Arrangements and Their Applications

Part III: Motion Planning and Related Applications

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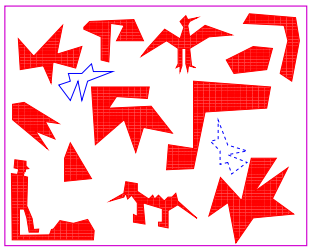
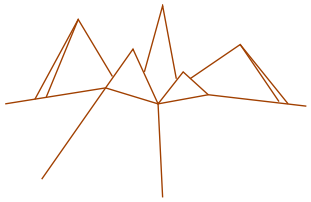
Configuration space

- arrangements $\mathcal{A}(\mathcal{S})$ are used for exact discretization of continuous problems
- a point p in configuration space $\mathcal{C}$ has a property $\Pi(p)$
- if a neighborhood U of p is not intersected by an object in $\mathcal{S}$, the same property $\Pi(q)$ holds for every point $q \in U$
  (the same holds when we restrict the configuration space to an object in $\mathcal{S}$)
- the objects in $\mathcal{S}$ are critical
- the property is invariant in each cell of the arrangement

Invariant, examples



collision-free motion
collides or not



representation of all the different 2D images of a polyhedral terrain
the scene graph

The basic motion planning problem

Let B be a robot system having k degrees of freedom and free to move within a two- or three-dimensional domain V which is bounded by various obstacles whose geometry is known to the system. The motion-planning problem for B is, given the initial and desired final placements of the system B, to determine whether there exists a continuous motion from the initial placement to the final one, during which B avoids collision with the known obstacles, and if so, to plan such a motion.

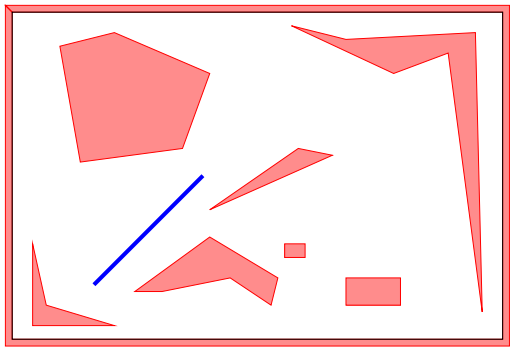
- its study is strongly tied with the study of arrangements
- many variants: moving obstacles, non-holonomy, uncertainty . . .

Outline of Part III

- introduction
- motion planning and arrangements
- probabilistic roadmaps vs. exact planners
- hybrid approach
- more applications of arrangements

Configuration space for translational motion planning

the rod is translating in the room



the reference point: the lower end-point of the rod
the configuration space is 2 dimensional

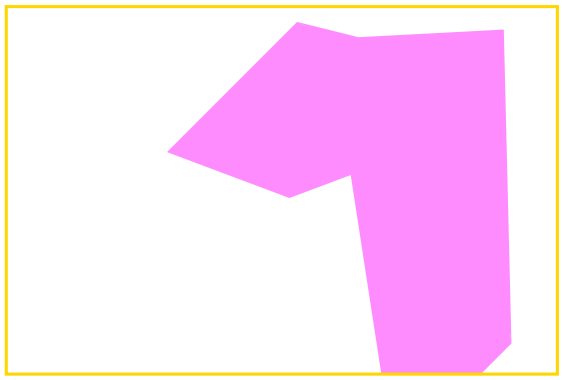
the dimension of the configuration space = the number of degrees of freedom of the moving system

Configuration space obstacles

the robot has shrunk to a point

$\Rightarrow$

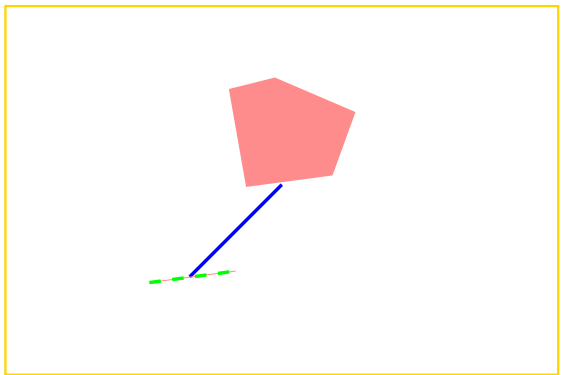
the obstacles are accordingly expanded



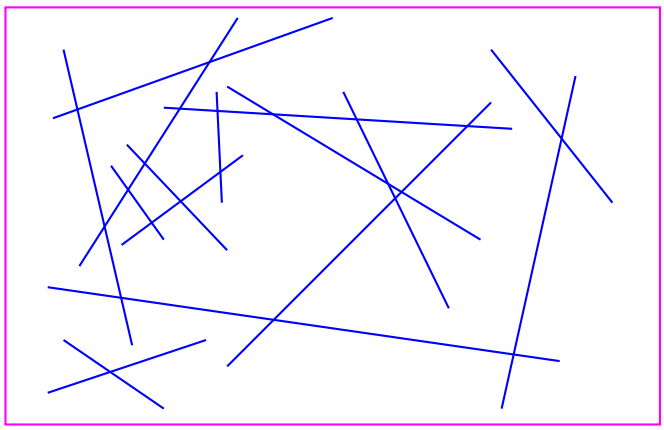
how to efficiently create expanded (C-space) obstacles?

Critical curves in configuration space

the locus of semi-free placements



The arrangement of critical curves in configuration space



suppose we know the initial configuration of the robot

More motion-planning configuration spaces

what is the dimension of the corresponding configuration space, and how do the critical surfaces look?

- a sphere moving among spheres in space
- a polyhedron translating among polyhedra in space
- a polygon translating and rotating among polygons in the plane
- industrial robot arms
- a polyhedron translating and rotating among polyhedra in space

General solutions to the motion-planning problem

- the problem is hard when k (# of dofs) is part of the input [Reif 79], [Hopcroft et al. 84], ...
- the Piano Movers series [Schwartz-Sharir 83], cell decomposition: a doubly-exponential solution, $O((nd)^{3^k})$
  assuming the robot complexity is constant, n is the complexity of the obstacles and d is the algebraic complexity of the problem
- roadmap [Canny 87]: a singly-exponential solution, $n^k(\log n)d^{O(k^2)}$ expected time

Near-optimal solutions for small k

- $k = 2$, near-linear
  - polygon robot translation [Edelsbrunner-Guibas-Sharir]
  - general [Guibas-Sharir-Sifrony 89]
- $k = 3$, near-quadratic
  - polyhedron robot translation [Aronov-Sharir 88]
  - polygon robot trans+rot [H-Sharir 93]
  - general [H-Sharir 94, Schwarzkopf-Sharir 96]
- $k \geq 4$
  - efficient solutions, suboptimal

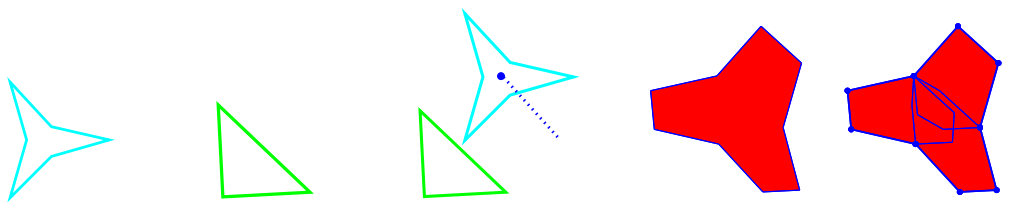
Exact motion planners, implementation

- polygonal translation w/degeneracies [Flato-H], based on CGAL arrangements
- polyhedral translation [Shaul-H]
- disc among polygons [Leiserowitz] based on
  - CGAL arrangements
  - with conic traits [Wein]
- on-going: planar translation and rotation

Minkowski sums and configuration space obstacles

given two planar sets A and B, their Minkowski sum $A \oplus B$ is the set $\{a + b \mid a \in A, b \in B\}$

the robot B collides with the obstacle A whenever the reference point of B is inside the sum $A \oplus (-B)$



Minkowski sums: algorithms

input: two polygonal sets P and Q, with m and n vertices

our algorithms have three steps:

(1) Decompose P and Q into convex subpolygons $P_1, P_2, \ldots, P_s$ and $Q_1, Q_2, \ldots, Q_t$

(2) compute all the Minkowski subsums $P_i \oplus Q_j$

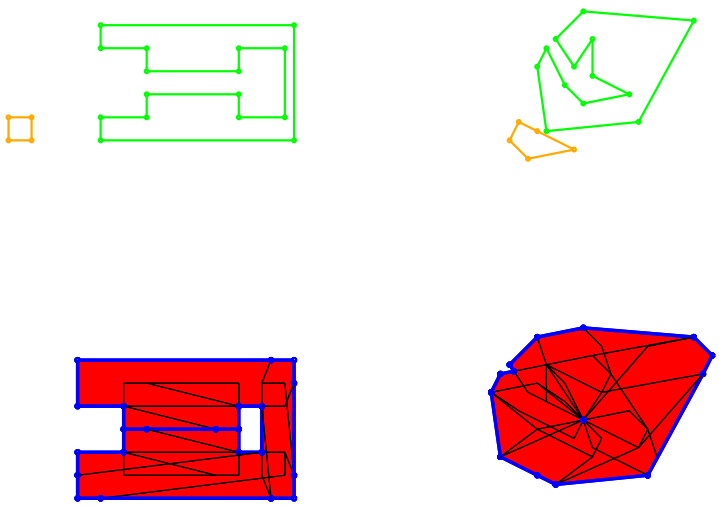
(3) compute the union of all the subsums $P_i \oplus Q_j$, using arrangements (planar maps)

in CGAL on top of the arrangements package, using filtered rationals

the algorithms differ in the convex decomposition (Step 1)

and the way they compute the union (Step 3)

Exact Minkowski sums



Minkowski sums: complexity

the Minkowski sum of two polygons with m and n vertices can have $\Omega(m^2n^2)$ vertices

they range in complexity from $\Theta(m+n)$ to $\Theta(m^2n^2)$

Minkowski sums: experiments

- different union algorithms: arrangement, incremental union, divide-and-conquer
- subsum insertion order: randomized, by “fatness”
- polygon decomposition has a crucial effect [Agarwal-Flato-H]: triangulations, min convex decompositions, mixed objective decompositions, heuristics

engineering challenge:
trapezoidal decompositions or triangulations are often costly in practice; design effective decompositions

Exact motion planners

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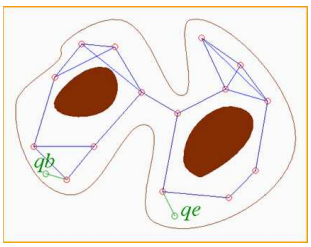
what about more degrees of freedom?

Meanwhile in robotics

- potential field methods [Khatib 80's]
  attractive potential (goal), repulsive potential (obstacles)
- random path planner [Barraquand-Latombe 89]
  combining PF with randomizationm to escape local minima
- probabilistic roadmaps
  [Kavraki-Svestka-Latombe-Overmars 95]

PRM: probabilistic roadmaps

- the approach



- easy to implement, provided you have a good static collision detector
- extended the applicability of motion planning: animation, docking motion, virtual prototyping

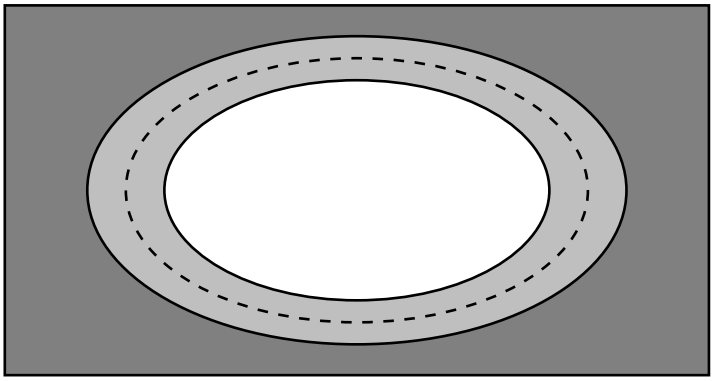
  the example of assembly maintenance

PRM, difficulties

- unpredictability
- narrow passages

Hybrid motion planning

combining exact arrangements with PRM

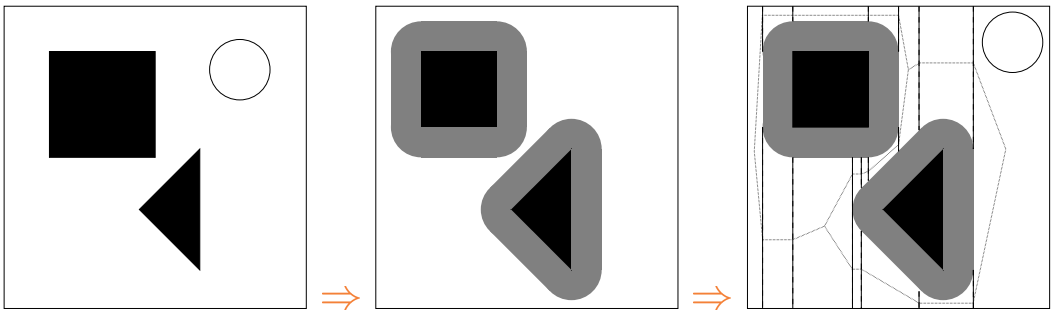


Hybrid motion planning

coordinating two discs moving among polygonal obstacles

[Hirsch-H 02]

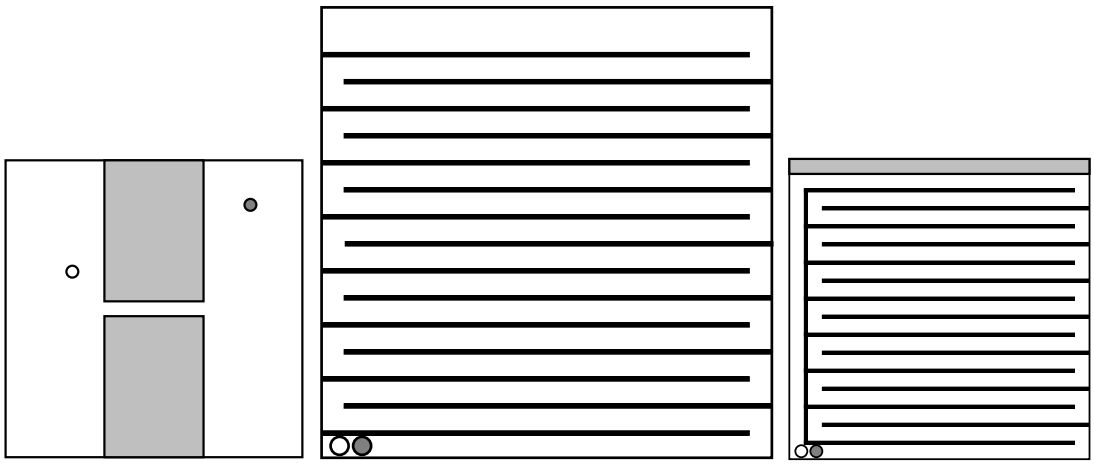
(1) exact arrangement for each disc



(2) Cartesian product of 2D free cells to create 4D cells

(3) PRM inside each 4D cell and between 4D cells

Hybrid motion planning, results



More applications of arrangements

- aspect graphs
- assmebly planning, the motion-space approach
- part orienting
- casting
- facility location
- . . .

References

- Robot Motion Planning, Latombe, Kluwer, 1991
- CRC Handbook (Goodman-O'Rourke) chapters on:
  - Algorithmic Motion Planning, Sharir
  - Robotics, Halperin-Kavraki-Latombe
- The surveys cited in the bibliography of Part I
- www.give.nl/movie/

Solutions to Problems 1–3

- **(1)** D. Halperin and M. Sharir. A near-quadratic algorithm for planning the motion of a polygon in a polygonal environment. *Discrete Comput. Geom.*, 16:121–134, 1996.
- **(2)** M. de Berg, D. Halperin, M.H. Overmars, and M. van Kreveld. Sparse arrangements and the number of views of polyhedral scenes. *Int. J. Comput. Geom. Appl.*, 7:179–195, 1997.
- **(3)** L.J. Guibas, D. Halperin, H. Hirukawa, J.-C. Latombe, and R.H. Wilson. Polyhedral assembly partitioning using maximally covered cells in arrangements of convex polytopes *Int. J. Comput. Geom. Appl.*, 8:179–199, 1998.

THE END of PART III