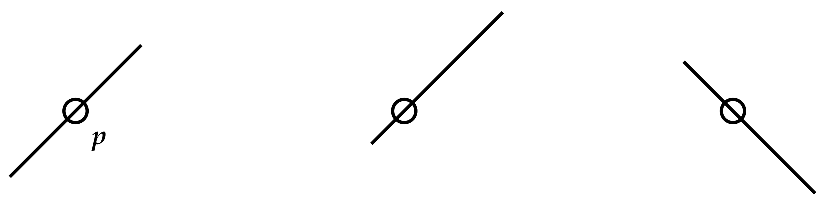
Assignment no. 2

Exercise 2.1 We are given a convex polygonal robot P with m vertices that is free to translate inside a convex polygonal room Q with n vertices. The only obstacles to the motion of P are Q's walls. What is the maximum combinatorial complexity of the free space in this case? Describe an efficient algorithm to compute it.

Exercise 2.2 Assume we have at our disposal an efficient algorithm to compute the Minkowski sum of two simple polygons. Use it to solve the following problem. We are given two overlapping simple polygons P and Q with m and n vertices respectively; find the minimum length translation of Q such that the interiors of P and Q become disjoint.

Exercise 2.3 A robot arm with two degrees of freedom consists of a line segment that passes through a point p in the plane. It can rotate around p and translate through p, but at all times it coincides with p. See the following figure for an illustration. The obstacles are pairwise interior-disjoint polygons



with a total of n vertices. Describe the constraint curves induced by this problem (the loci of semi-free configurations). What is the maximum combinatorial complexity (give asymptotic lower and upper bounds) of the free space of this motion-planning problem. Give an efficient algorithm to plan a collision-free motion from given start to goal configurations if such a motion exists.

Exercise 2.4 A planar scene consists of n pairwise disjoint line segments. Describe the critical curves of the aspect graph of the scene (i) under orthographic and (ii) under perspective projection (in flatland). How complex can the arrangements underlying these graphs be?

Next, address the same questions for a scene consisting of n pairwise disjoint triangles in 3-space (where the viewpoints are in space as well).

Exercise 2.5 A rigid L-shaped robot (made of two line segments perpendicularly attached) moves among n point obstacles in the plane. Describe the configuration-space arrangements of constraints and bound their worst-case complexity when (i) the robot translates only, and (ii) the robot translates and rotates.