I Erdős Magic

Sixty years ago Paul Erdős started a methodology to prove the existence of mathematical objects. In modern terms, a randomized algorithm is given that may create the desired object. If one can prove the algorithm has a positive probability of success then the object *must* exist. We examine several examples:
(i) finding an independent set in a graph. (Idea: Greedy Algorithm)
(ii) given sets $A_1, \ldots, A_n$ finding (not always possible!) a two coloring of the underlying points so that no A_i is monochromatic. (Idea: Color Randomly)
(iii) given n sets $A_1, \ldots, A_n$ on n vertices find a two-coloring of the underlying points so that for each A_i the "discrepency," the difference between the number of points in A_i in the two colors, is small. (Idea: Color Randomly)
(iv) Find n points in the unit square so that none of the triangles formed by any three of the points is "too small." (Idea: Throw down points at random but then appropriately "modify".) We further discuss "derandomization," replacing the Erdős-type argument with an explicit and rapid algorithm.

II The Erdos-Renyi Phase Transition

Some forty five years ago Paul Erdős and Alfred Rényi wrote "On the Evolution of Random Graphs." We begin with a general discussion of the random graph $G(n, p)$, having n vertices and probability p of adjacency. Erdős and Rényi recognized that the random graph $G(n, p)$ undergoes a fundamental change when $p \sim \frac{1}{n}$. Parametrizing $p = \frac{c}{n}$, while $c < 1$ all components are small and simple but when $c > 1$ a complex giant component has emerged. Today we recognize this as a phase transition. Phase transitions (= sudden change, e.g., freezing) appear in mathematical physics (e.g., bond percolation on Z^d), computer science (e.g., random k-SAT) and other places and we give a general discussion of them. For Erdős-Renyi percolation we can expand the $c = 1$ value and we explain why the "proper" parametrization for the "critical window" is $p = n^{-1} + \lambda n^{-4/3}$.

We explore this percolation phenomenon from a variety of viewpoints. One new approach (joint with Remco van der Hofstad) involves a novel analysis of the Breadth First Search algorithm on the random graph $G(n, p)$.