

Cost-sharing methods.

Lecture 2. Exercises

1. Prove cross-monotonicity of KLS
2. Show an example where the total dual constructed from Lifted Cut relaxation recovers $\frac{1}{2}$ the optimum. Hint: Same example works for undirected cut relaxation.
3. Show that cost-monotonic cost shares provide an allocation in the core.
4. Modify the lower bound construction for Steiner tree to obtain a lower bound of 3 for Facility location. Work out the high level idea of the lower bound. Hint: place facilities of cost 3 at vertices of f_B , remove connections to the root.
5. Give a simple proof of 2-budget balance for KLS on Steiner tree. Hint: Pair each terminal with a terminal located at the root.
6. Argue that the best Steiner tree apx (1.55) can be used in the Subproblem step of SROB without harming 2-strictness
7. Use Sample-augment to provide exponential time 4-budget balance cross-monotonic cost shares for SROB. Hint: Define cost-shares as the expected value of cross-monotonic cost shares for the Steiner tree constructed on sampled demands. Argue that the expected connection cost is also cross-monotonic.
8. Show that AKR provides 2-strict cost-shares for Steiner tree. Hint: Pair each terminal with a terminal located at the root.