Coordination Mechanism and Search Problems **ADFOCS 2009**

Exercise 1: September 14, 2009

1. Consider the Makespan policy coordination mechanism for scheduling in the restricted assignment model on m machines (i.e. all jobs in a machine are delayed until all of them were completed). The goal of the system is to minimize the maximum completion time. The goal of the job is to minimize its own completion time.

   (a) Show that the price of anarchy of deterministic NE is $O(\log m / \log \log m)$. Note that in class we only proved it for jobs of size 1 and optimal value of 1.
   Hint: follow similar structure of the proof in class.

   (b) Assume that all jobs are of the same size which is at most $1/(4 \log m)$ of the optimal maximum completion time. Show that the price of anarchy of deterministic NE is at most 2.

2. Consider scheduling on m related machines. The goal of the system is to minimize the maximum completion time. The goal of the job is to minimize its own completion time.

   (a) Assume that all the speeds of the machines are real numbers between 1 and 2. Show that the price of anarchy of deterministic NE for the coordination mechanism that uses the I.D. priority policy is a constant.

   (b) Show the same to the Makespan policy.

   (c) Assume that all the speeds of the machines are arbitrary but all jobs have sizes which are between 1 and 2. Show that the price of anarchy of deterministic NE for the coordination mechanism that uses the I.D. priority policy is a constant.

   (d) Show the same to the makespan policy.

3. Consider scheduling on m machines in the hierarchy model which means that the set of machines which is associated with a job i is $[1 \ldots k_i]$ for some $1 \leq k_i \leq m$. The goal of the system is to minimize the maximum completion time. The goal of the job is to minimize its own completion time.

   (a) Show that the price of anarchy of deterministic NE for the Makespan policy is not a constant (i.e. it depends on m).

   (b) Show that the price of anarchy of deterministic NE for the I.D. priority policy is not a constant (i.e. it depends on m).

   (c) Design a coordination mechanism whose price of anarchy for deterministic NE is a 2.
   Hint: a machine knows the value of k_i for each job i assigned to it.

4. Consider scheduling in the restricted assignment model on m machines where both the algorithm and the optimum are allowed to split jobs (in any way). The goal of the system is to minimize the maximum completion time. The goal of the job is to minimize its own completion time.

   (a) Show a lower bound of $H_m = 1 + 1/2 + \ldots + 1/m$ on the price of anarchy of deterministic NE for the coordination mechanism that uses the I.D. priority policy.

   (b) Show the same lower bound for the Shortest First policy (shortest means with respect to the size of the part of the job assigned to the machine).

   (c) Prove that the price of anarchy of the Makespan policy is 1.

5. Consider scheduling in the restricted assignment model on m machines. Each machine uses I.D. priority policy to order the jobs but the priority orderings of the various machines may be different.The goal of the job is to minimize its own completion time.

   (a) Show that there is deterministic NE in this setting.
   Hint: define a potential on the states and show that if a job can improve the potential will be reduced.

   (b) Show that for unrelated machines in the above setting, deterministic NE may not exists.
   Hint: There is such an example already with 2 machines and 3 jobs.

Exercise 2: September 16, 2009

1. Consider scheduling on m unrelated machines. The goal of the system is to minimize the maximum completion time. The goal of the job is to minimize its own completion time. Show that the price of anarchy of deterministic NE for the coordination mechanism that uses the I.D. priority policy is at most m (and, in particular, Shortest First is at most m).

2. Consider scheduling on (even) m machines which consists of two groups of $m/2$ identical machines. For each i, job i has weight w_i if it is assigned to a machine from the first group and possibly a completely different weight p_i if it is assigned to a machine from the second group. The goal of the system is to minimize the maximum completion time. The goal of the job is to minimize its own completion time.

   (a) Show that the the price of anarchy of deterministic NE for the Makespan coordination mechanism is unbounded even as a function of m.

   (b) Design a coordination mechanism whose price of anarchy for deterministic NE is a constant (independent of m).

   (c) Is price of anarchy of Shortest First policy a constant ?

3. Consider an on-line routing problem on a graph $G = (V, E)$. Request i consists of (s_i, t_i, p_i). To serve the request i you need to allocate a path from s_i to t_i with load p_i. The load on edge e is the sum of p_i for all i whose allocated path uses the edge e.

   Design an $O(p)$ competitive algorithm for minimizing the L_p norm of the load vector of the edges. From that you can conclude an $O(\log |E|)$ competitive algorithm for minimizing the maximum load over the edges.
   Hint: follow the algorithm and the proof of the online scheduling on unrelated machines.

4. Show a lower bound of $\Omega(p)$ for the competitive ratio of any online deterministic algorithms for online scheduling on unrelated machines.
   Hint: the lower bound is valid even for the restricted assignment model.

5. Consider the problem of on-line scheduling of jobs on kr machines that are partitioned into k groups, each consists of r identical machines. Job i has a weight vector of length k were w_{ij} (for $1 \le j \le k$) is the processing time of a job on a machine if it is assigned to a machine in the group j.

   (a) Show an $O(\log k)$ competitive algorithm for minimizing the maximum load over all machines.
   Note: you may need to use also doubling.

   (b) Show a lower bound for minimizing the maximum load over all machines of $\Omega(\log k)$ for any k and r.