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Exercises for ADFOCS 2018 - Sheet 1

Exercise 1 **Longest Palindrome Subsequence Problem**: Given a sequence S of length n, find the longest subsequence which is a palindrome (i.e., a sequence of characters which reads the same backward and forward).

Prove that if this problem can be solved in time $O(n^{2-\varepsilon})$ then **OVH** fails.

Exercise 2 **Diameter Problem**: Given a graph G on n vertices and m edges, compute the largest distance between any two vertices in G.

We consider sparse graphs, i.e., $m = \widetilde{O}(n) = O(n \operatorname{polylog} n)$. Show that the diameter can be computed in time $\widetilde{O}(n^2)$, and prove that if the diameter can be computed in time $O(n^{2-\varepsilon})$ then **OVH** fails.

Exercise 3 **k-Clique Problem**: Given a graph G on n vertices, decide whether there are vertices $v_1, \ldots, v_k$ that are pairwise adjacent.

Show by a reduction that if **OVH** fails, i.e., OV can be solved in time $O(n^{2-\varepsilon}\text{poly}(d))$, then for sufficiently large k and some $\varepsilon' > 0$ the **k-Clique** problem can be solved in time $O(n^{k-\varepsilon'})$.

Remark: The fastest known algorithm for k-Clique runs in time $O(n^{k\cdot\omega/3})$ where $\omega \leq 2.37$, so this reduction does not yield a tight lower bound for OV, but is only a partial relation.

Exercise 4 **RegExpMatching**: Given a regular expression R of size m and a text T of length n, determine whether any substring T' of T can be derived from R.

It is well-known that this problem can be solved in time $O(nm)$. Show that there is no algorithm running in time $O((mn)^{1-\varepsilon})$ unless **OVH** fails.

For *specific classes* of regular expressions there are faster algorithms to solve this problem. Consider *homogeneous regular expressions*: A regular expression R is called *homogeneous of type "$o_1 o_2 \ldots o_\ell$"* (where $o_i \in \{\circ, *, +, |\}$) if there exist $a_1, \ldots, a_p$, which are characters or homogeneous regular expressions of type $o_2, \ldots, o_l$, such that $R = o_1(a_1, \ldots, a_p)$. For example, the regular expression $[(a \circ b \circ c) \,|\, b \,|\, (a \circ b)]^*$ is homogeneous of type "$* \,|\, \circ$", but the regular expression $(a^*) \,|\, (b^+)$ is not homogeneous.

a) Find types t such that **RegExpMatching** restricted to homogeneous regular expression of type t can be solved in time $O(n + m)$.

b) Prove that there is no $O((mn)^{1-\varepsilon})$ algorithm for **RegExpMatching** restricted to homogeneous regular expression of type "$| \circ |$" unless **OVH** fails.
Prove the same result for homogeneous regular expressions of type "$| \circ *$".