ADFOCS: Market Design
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Problem Set #1

1. Consider the following market. There are 3 schools with one seat each. There are two students, Alice and Bob, with values of $10, 2$ and 0 for schools $1, 2$ and 3, respectively. There is one student, Carol, with value $7, 5$ and 0 for schools $1, 2$ and 3, respectively.

   - Calculate the expected value of each student in the random serial dictatorship mechanism (RSD).
   - Describe a different lottery that improves each student's expected value.
   - Consider the following raffle mechanism: each student has a raffle ticket and can place it into the "bucket" of any school. Each school admits one student, chosen randomly from among the students who put tickets in its bucket. Calculate all equilibria (i.e., bucket choices of students that maximize each student's expected value, holding fixed the choices of others). What is the resulting lottery?

2. Consider the following market. There are n schools. School j has 2^{j-1} seats. There are n types of students with 2^n students of each type (so $n2^n$ students in total). Students of type i have value $v_{ij} = 1 + 2^{-100j}$ for schools $j, 1 \leq j \leq i$, and $v_{ij} = 2^{-100j}$ for schools $j, i < j \leq n$.[1]

   - Calculate the expected value of a student of type i in the random serial dictatorship mechanism (RSD).
   - Describe a different lottery that improves each student's expected value by a factor $n/2$.

   Thus, while the efficiency of RSD can degrade with the size of the market, one can prove the raffle mechanism from question 1 is approximately efficient (i.e., there is no alternate lottery that improves each student's expected value by a constant).[2] This comes at a cost: the raffle mechanism is not strategyproof as demonstrated in question 1.

3. Suppose agents have *endowments* as might be the case in dormitory assignment. There are n students and n rooms; student i is currently assigned room r_i. Students have strict preferences over rooms. We focus on individually rational (IR) outcomes: i.e., each students must prefer her assignment in the outcome to her endownment.

   - Suppose all students prefer r_i to r_j whenever $i < j$. What is the unique IR outcome?
   - Suppose $n = 3$. All students prefer r_1. Student 2 prefers r_3 to r_2 whereas students 1 and 3 prefer r_2 to r_3. What is the unique IR outcome?

[1] The point of the 2^{-100j} term in the values is to give all students the same total ordering over the options; you can ignore this term in the welfare calculations.

[2] See N. Immorlica, B. Lucier, J. Mollner, and E.G. Weyl, *Approximate Efficiency in Matching Markets.*

- Consider the following mechanism: Let S be a subset of students. Set S equal to the set of all students. While $S \neq \emptyset$ (i.e., students remain), each student $i \in S$ points to the student $j \in S$ whose room she likes best (i.e., $r_j \succ_i r_{j'}$ for all $j' \in S$).[3] For each cycle of students, assign each student in that cycle to the room of the student they are pointing to, and remove them from S.
  - Argue this mechanism terminates after at most n steps.
  - Argue this mechanism is IR.
  - Argue this mechanism is strategyproof.

In fact, while we won't discuss it here, this mechanism is *group strategyproof.* That is, no group of agents can all weakly improve by coordinating their strategies.

[3]Note students may point to themselves.