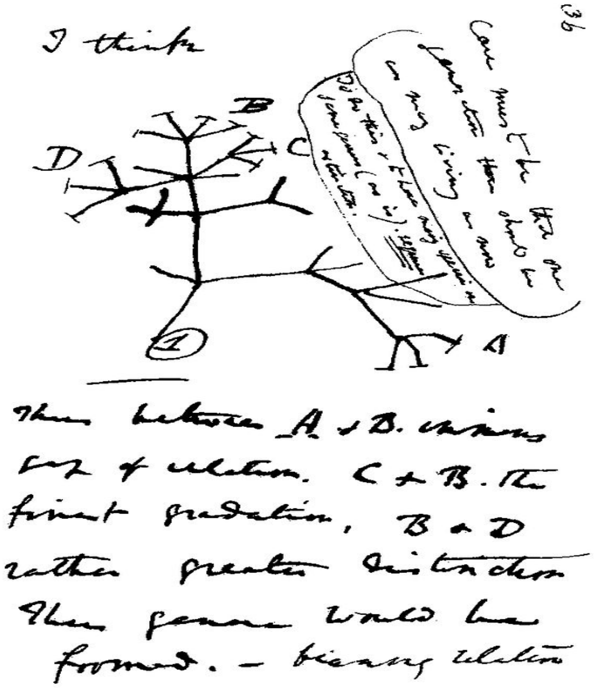
Fitting Tree Metrics with Minimum Disagreements

Evangelos Kipouridis

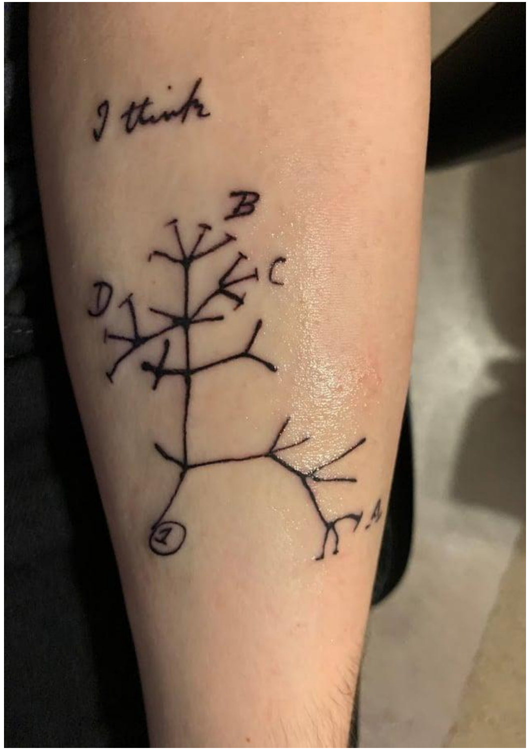
UNIVERSITÄT
DES
SAARLANDES

max planck institut
informatik

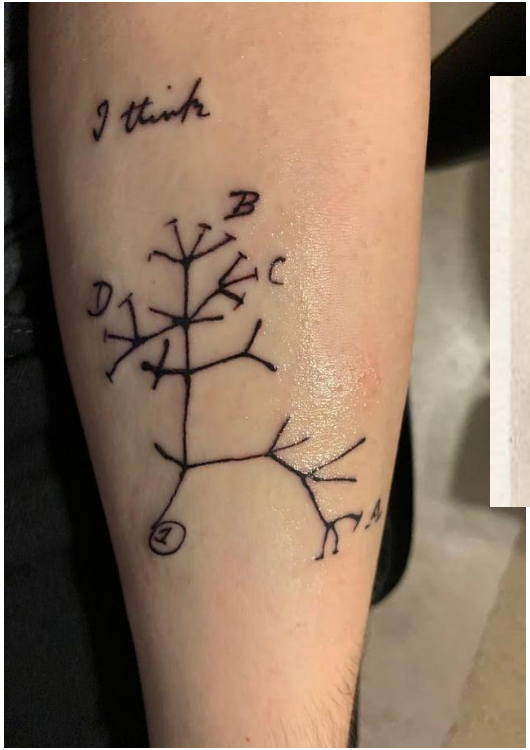
Any guess?

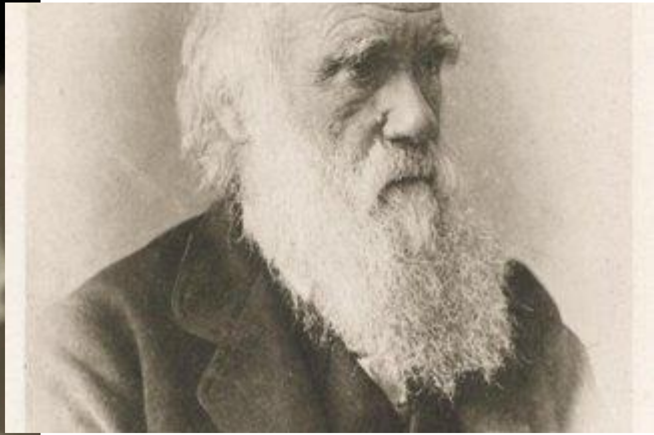

I think
B
C
D
1
A
Thus between A & B. immense gap of relation. C & B. the finest gradation, B & D rather greater distinction Thus genera would be formed. — bearing relation

Any guess?

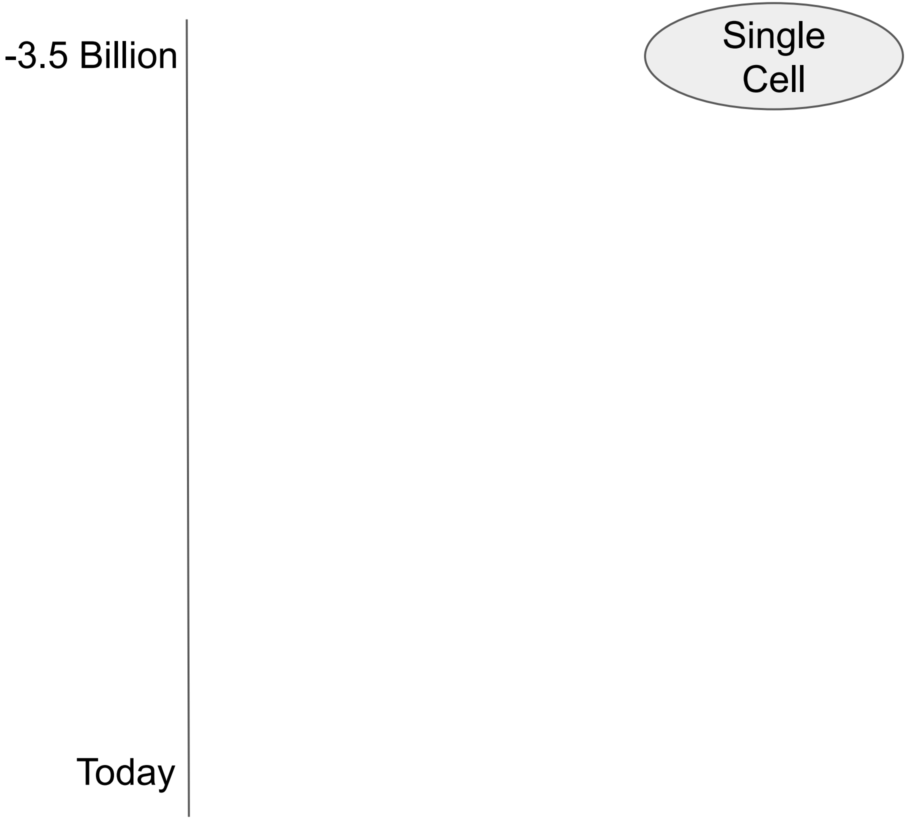


Darwin’s notes





Tree of life



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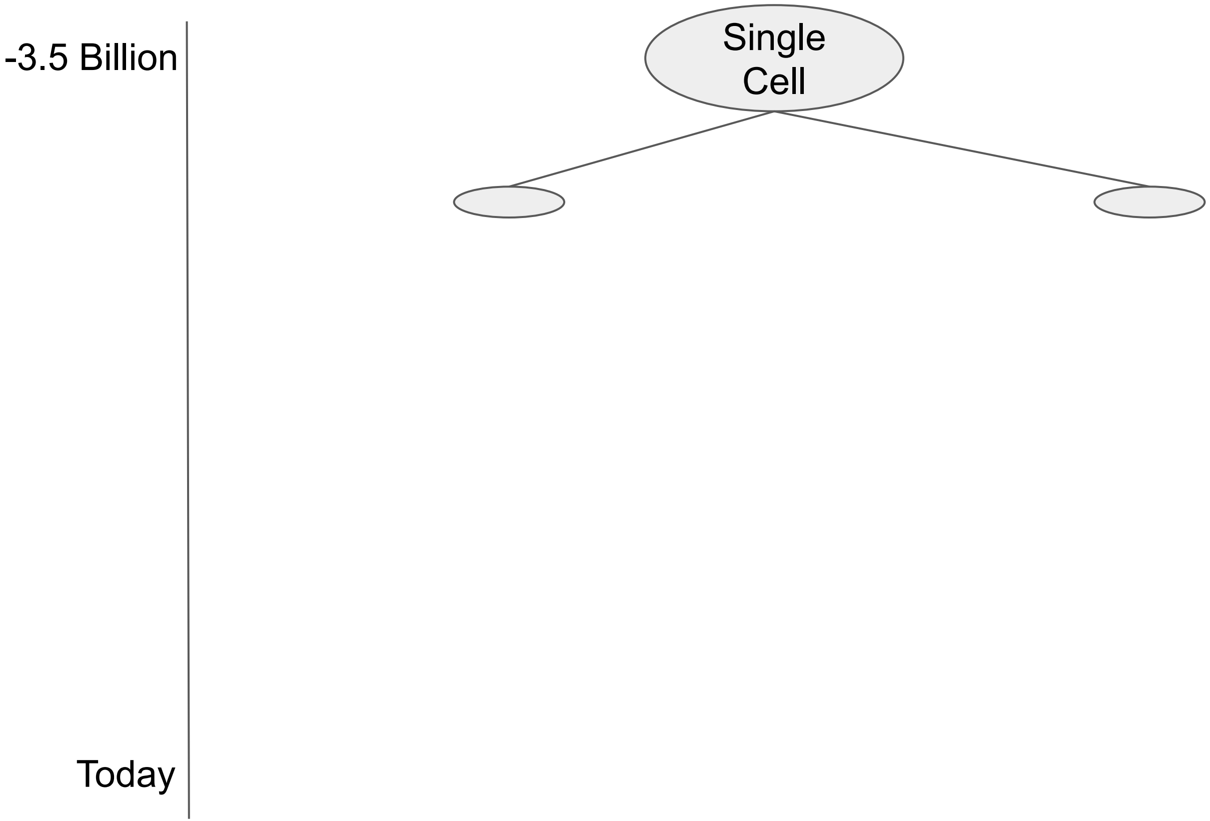

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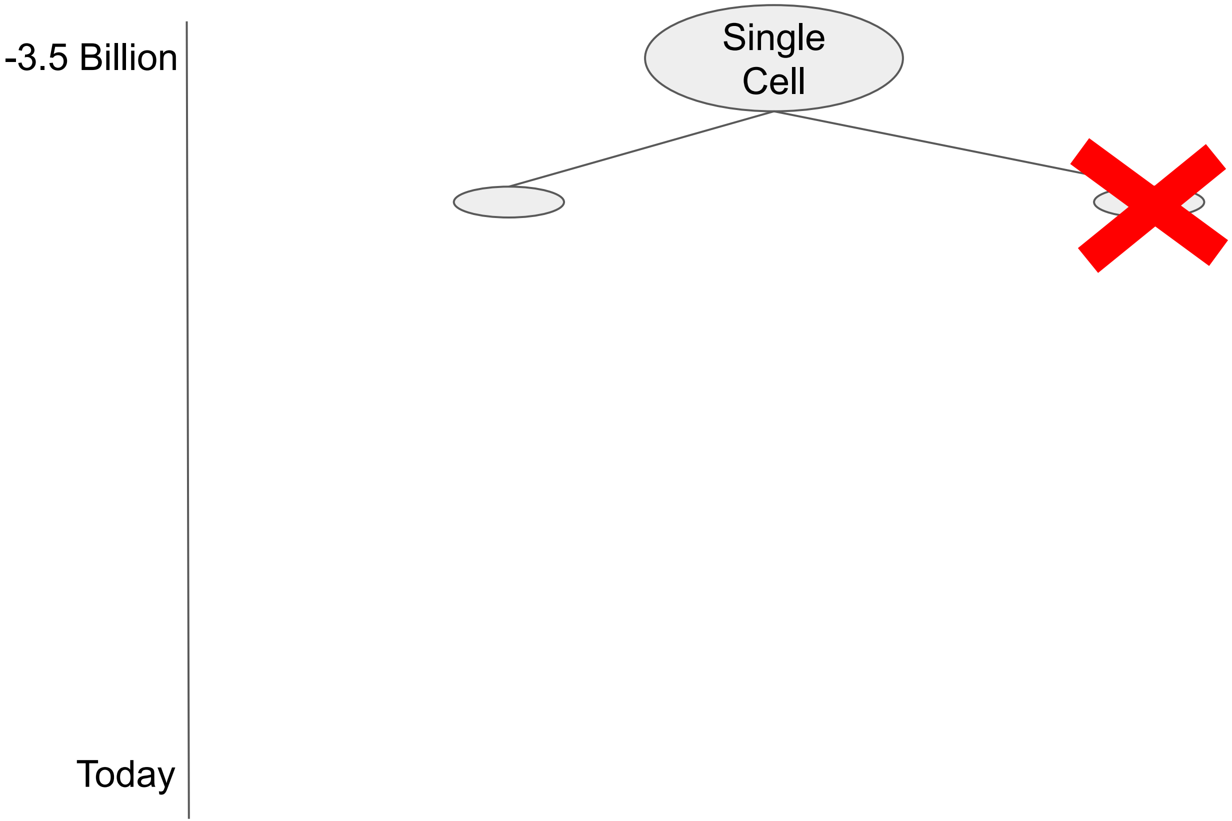

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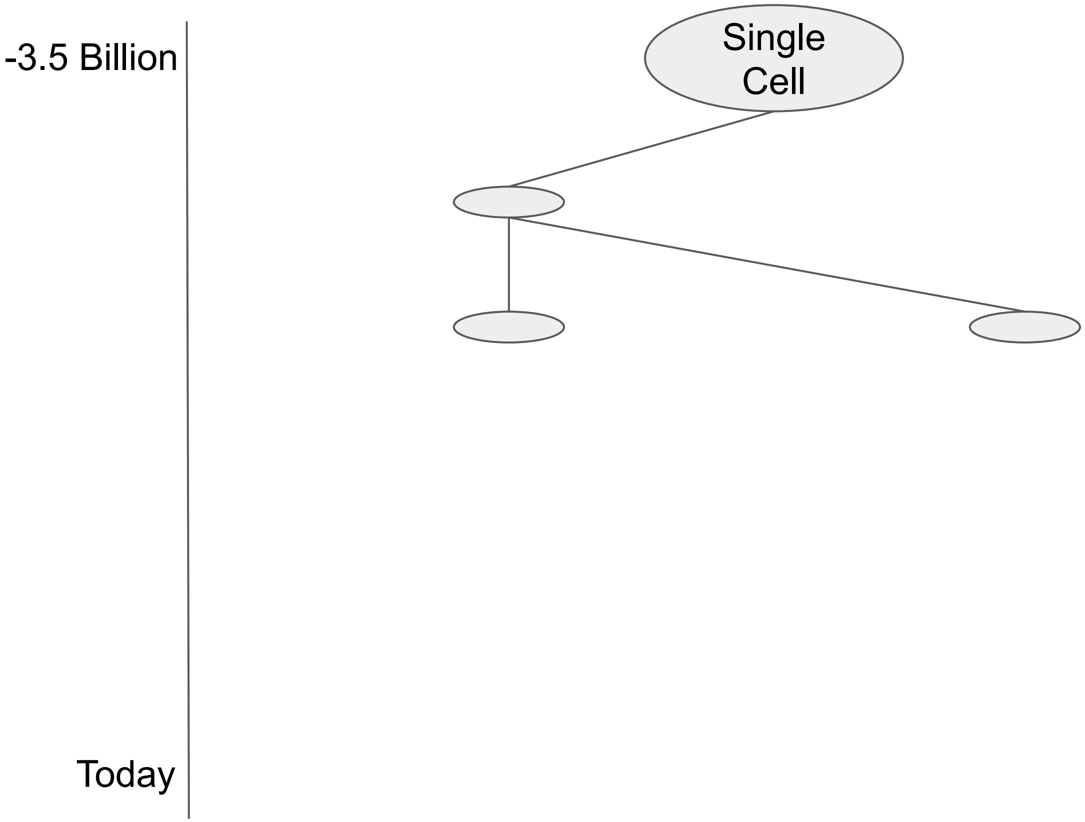

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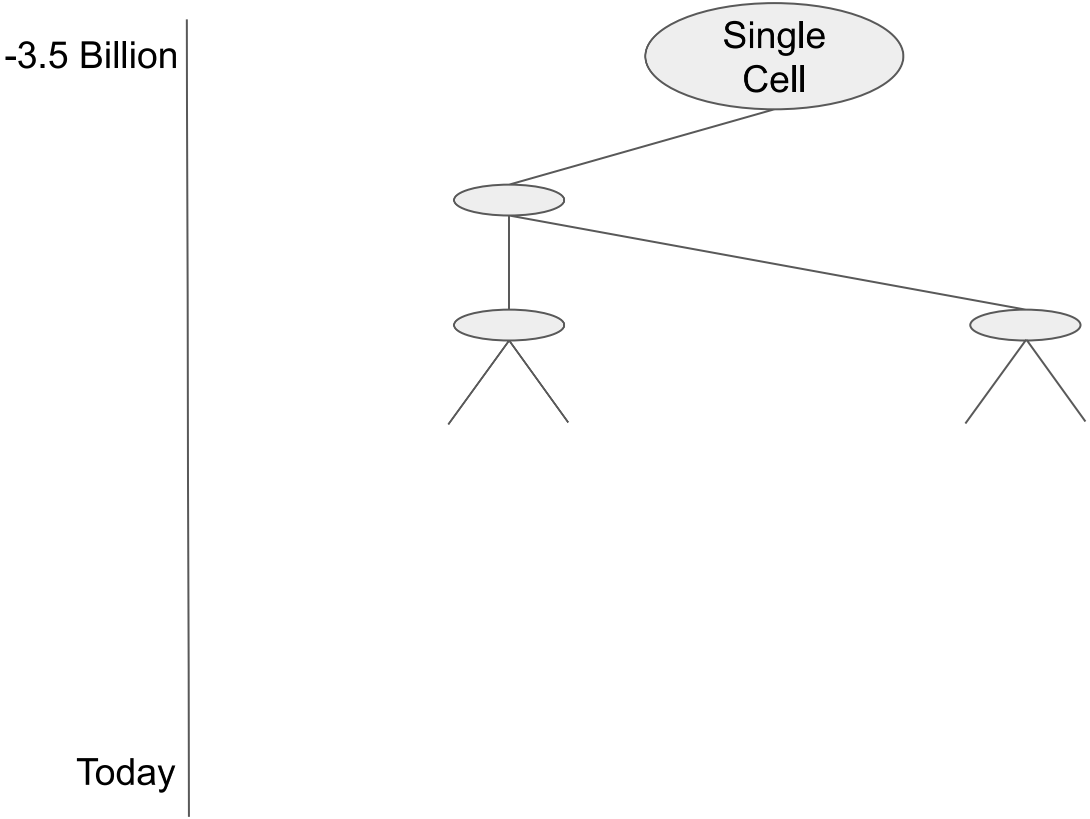

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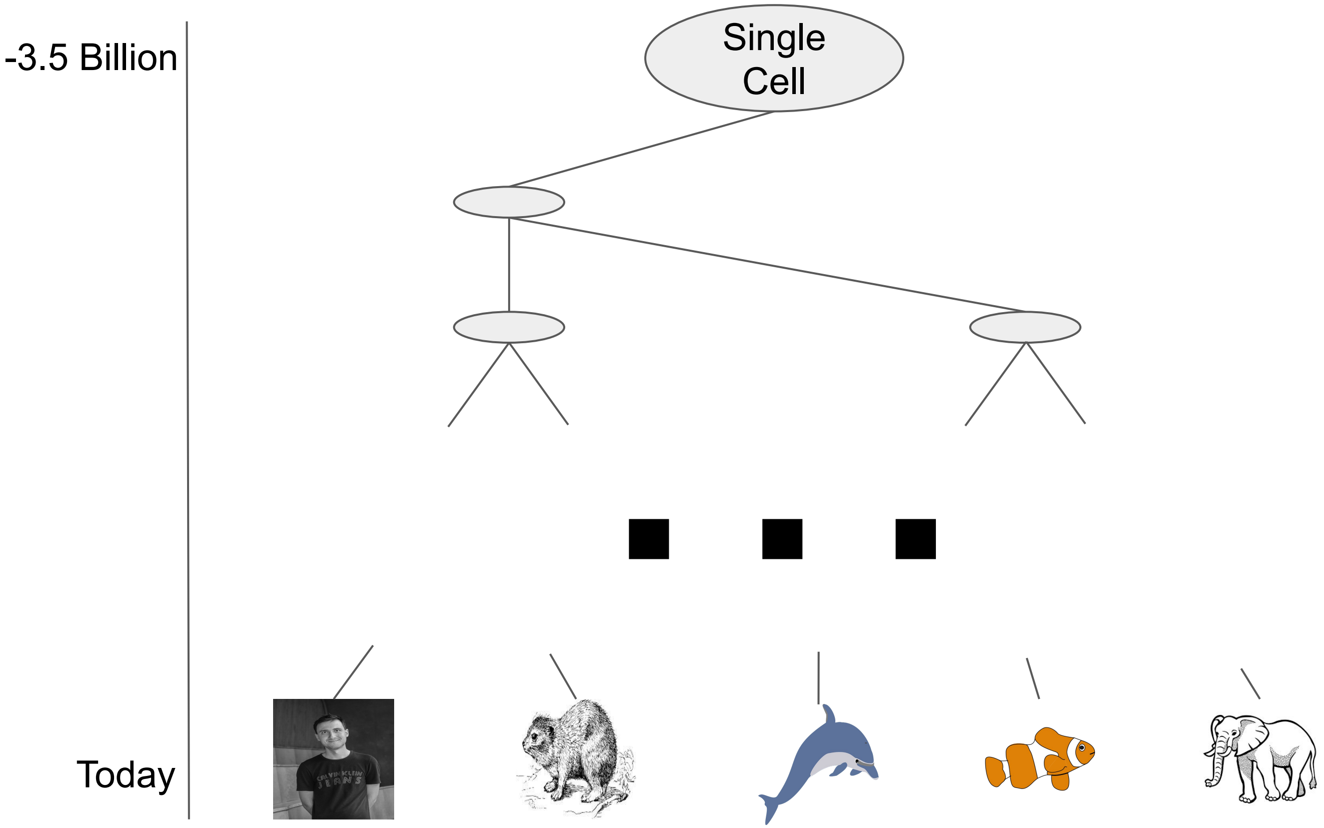
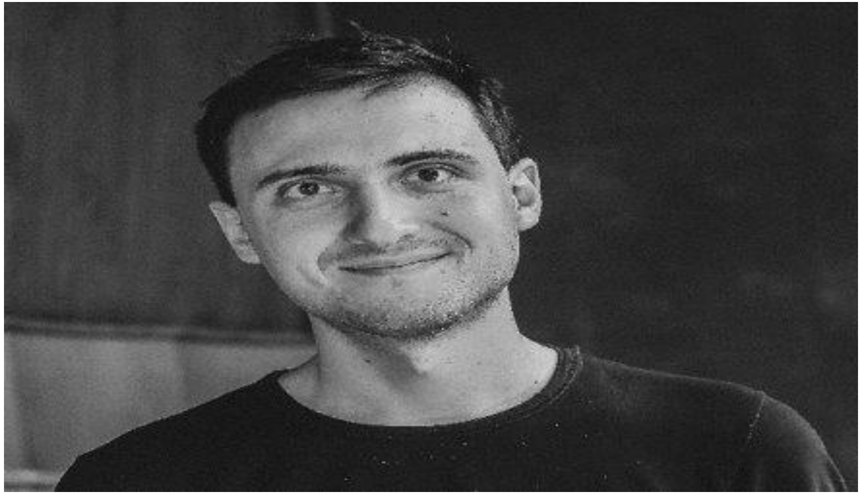


First to discuss tree reconstruction

They're the same picture…



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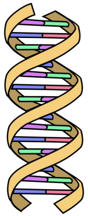

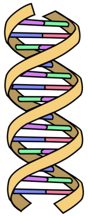

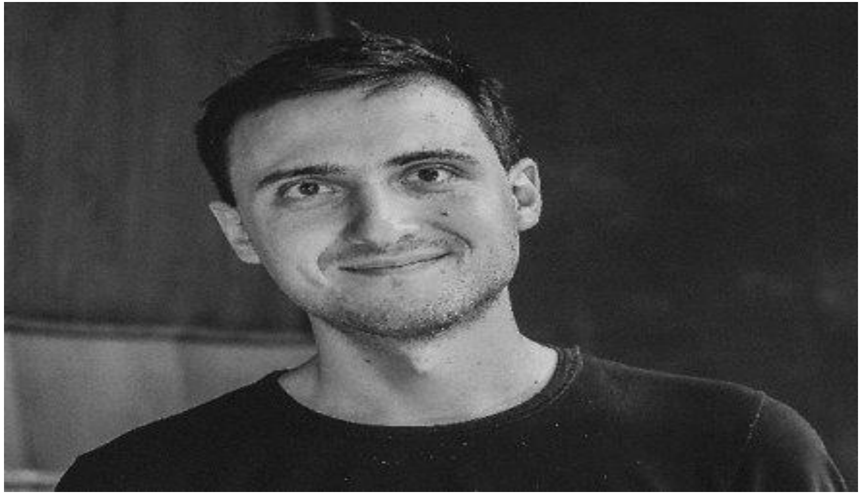

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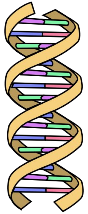
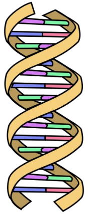
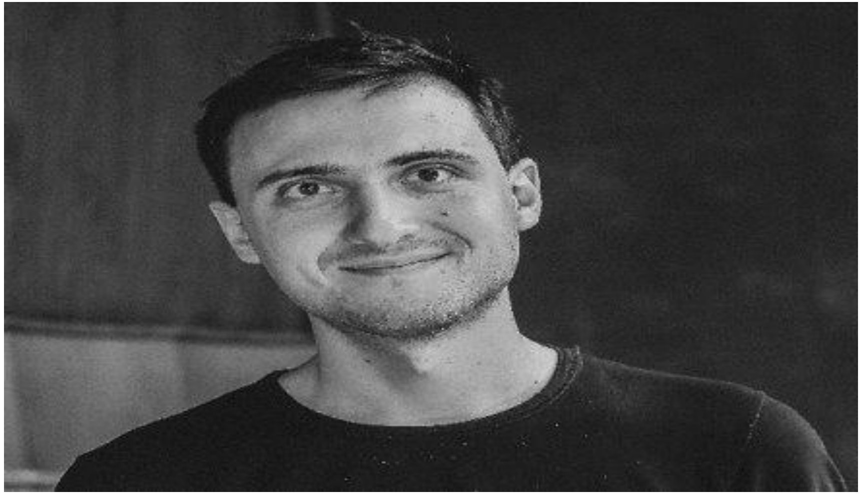


Hmm…
15 million years?





Reconstruct from distance matrix

				
	0	15	435	432
	15	0	436	433
	435	436	0	63
	432	433	63	0

Reconstruct from distance matrix

				
	0	15	435	432
	15	0	436	433
	435	436	0	63
	432	433	63	0

If, due to noise,
no matching tree?

Reconstruct from distance matrix

				
	0	15	435	432
	15	0	436	433
	435	436	0	63
	432	433	63	0

If, due to noise,
no matching tree?

Minimize
disagreements!

Reconstruct from distance matrix

				
	0	15	435	432
	15	0	436	433
	435	436	0	63
	432	433	63	0

If, due to noise,
no matching tree?

Minimize disagreements!

Can also minimize total error, max error, L2 error…

The New York Times

TRILOBITES

A Battle Is Raging in the Tree of Life

Which came first, the sponge or the comb jelly?

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Science > Animals

Scientists Have Found the First Branch on the Tree of Life

Something had to diverge from the trunk eventually.

BY JACKIE APPEL PUBLISHED: JUN 8, 2023

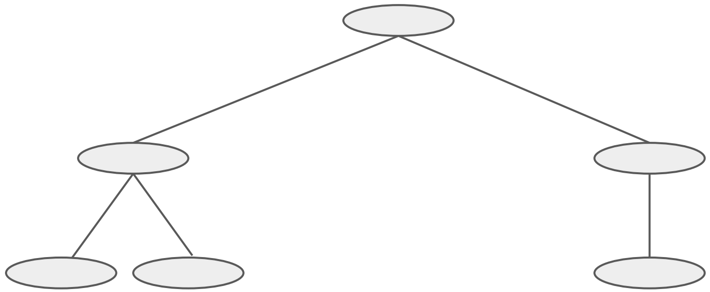
What we know…

APX - Hard

What we know…

APX - Hard

O(1) approximation for ultrametrics (structured trees) - even under mild constraints

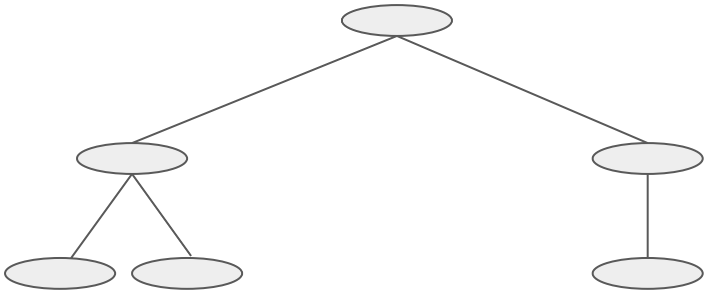


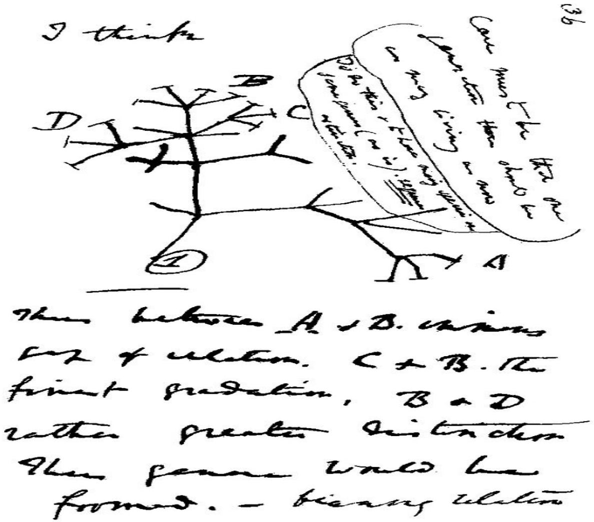
What we know…

APX - Hard

O(1) approximation for ultrametrics (structured trees) - even under mild constraints

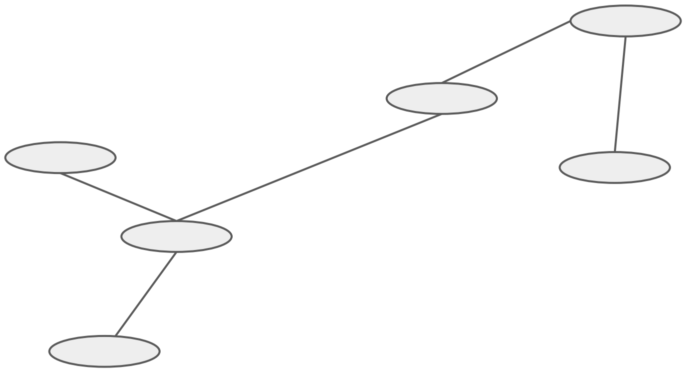
- What about unstructured trees?





Structuring the unstructured

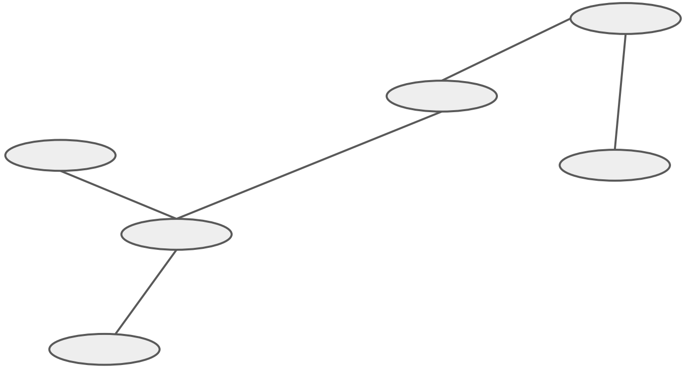
OPT



Structuring the unstructured

1) Find a root.
2) Find depths of leaves.

OPT



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1) **Find a root.**
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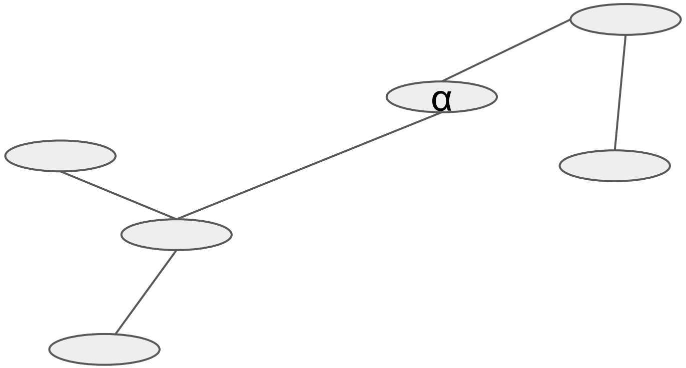

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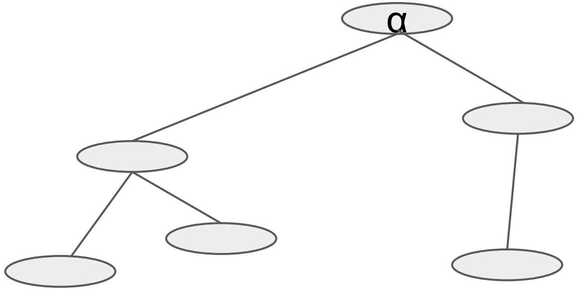

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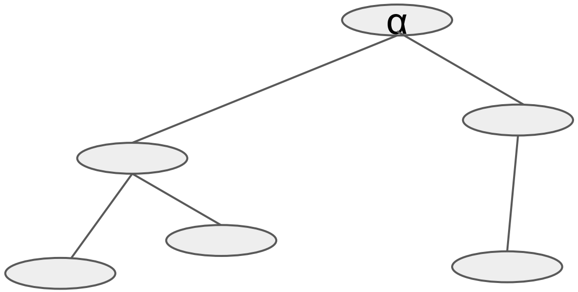

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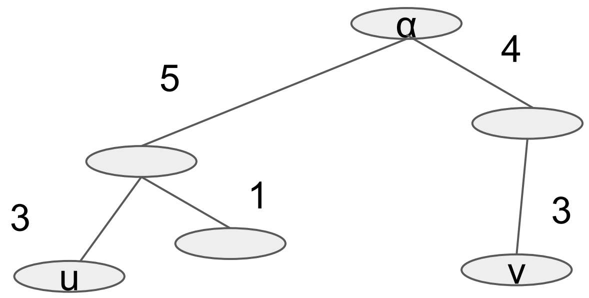
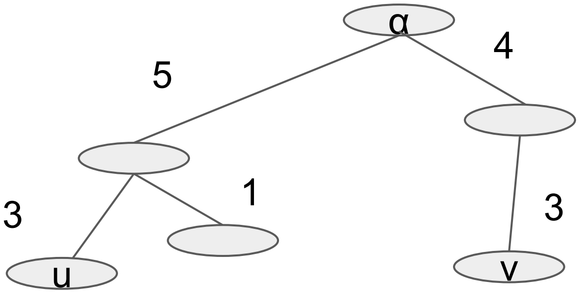


Structuring the unstructured

1) Find a root.
2) **Find depths of leaves.**

OPT

Input(α,u) = 12, but OPT(α,u) = 8

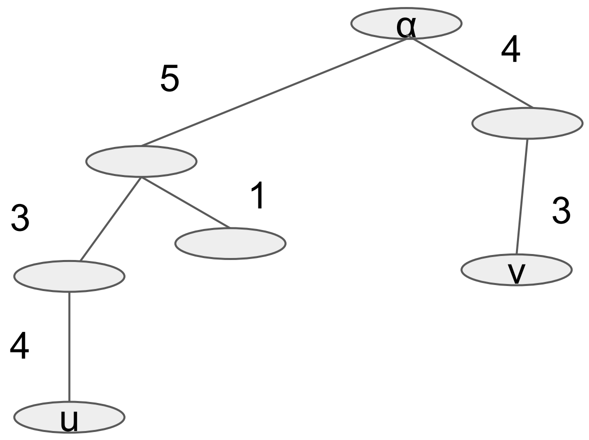


Structuring the unstructured

1) Find a root.
2) **Find depths of leaves.**

OPT’

Input(α,u) = 12, but OPT(α,u) = 8

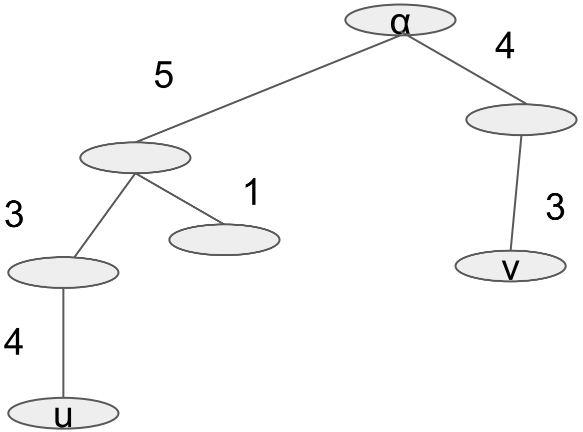


Structuring the unstructured

1) Find a root.
2) **Find depths of leaves.**

OPT'

Input(α,v) = 6, but OPT(α,v) = 7



Structuring the unstructured

1) Find a root.
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OPT'

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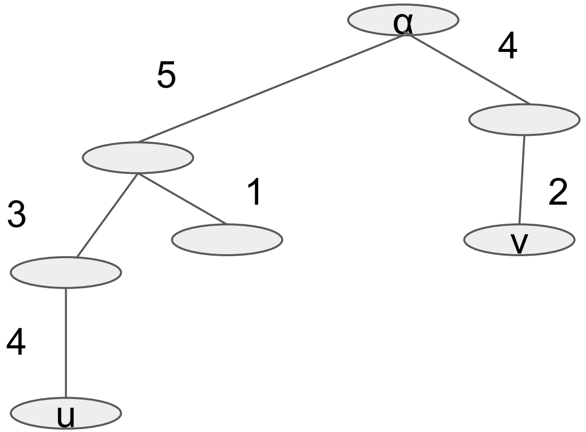
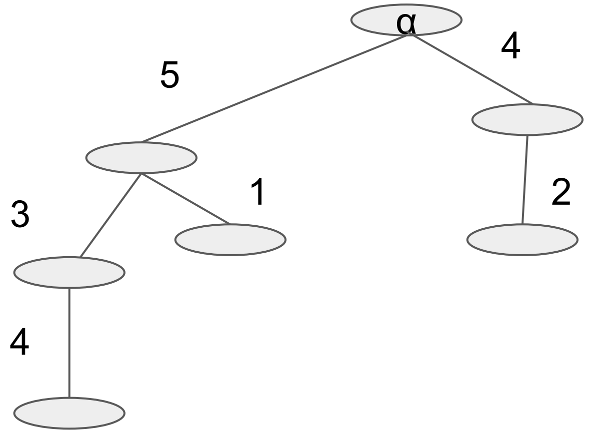


Structuring the unstructured

1) Find a root.
2) **Find depths of leaves.**

OPT'

Now for all u we know depth(u).
depth(u) = OPT'(α,u) = Input(α,u)

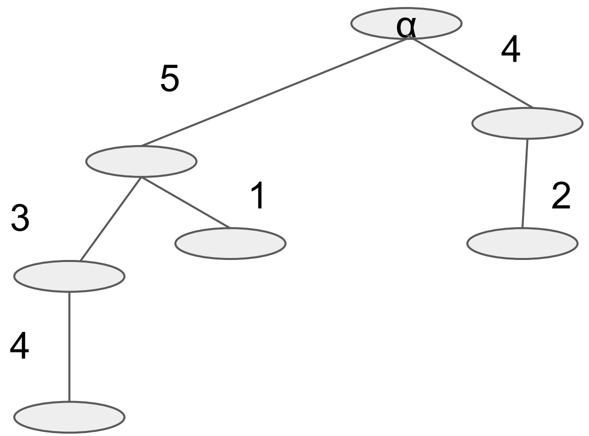


D() denotes disagreements in OPT

Structuring the unstructured

1) Find a root.
2) **Find depths of leaves.**

OPT'



Now for all u we know depth(u).
depth(u) = OPT(α,u) = Input(α,u)

How much did we pay?

- We moved exactly D(α) nodes, each introduced at most (n-1) disagreements.
- D(OPT') ≤ D(OPT) + D(α) (n-1)

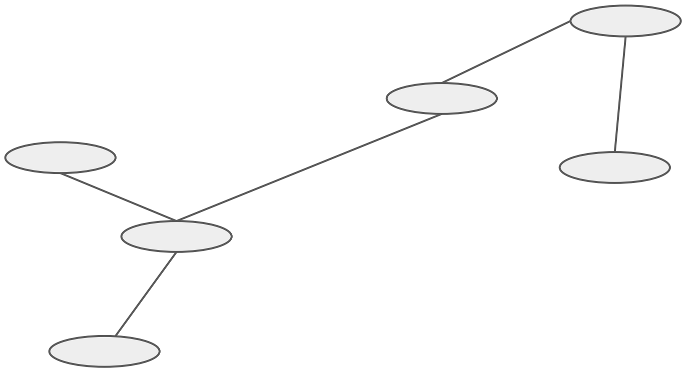
D() denotes disagreements in OPT
D(OPT') ≤ D(OPT) + D(α) (n-1)

Structuring the unstructured

1) **Find a root.**
2) Find depths of leaves.

$$D(OPT) = \frac{1}{2}\sum_u D(u)$$

OPT



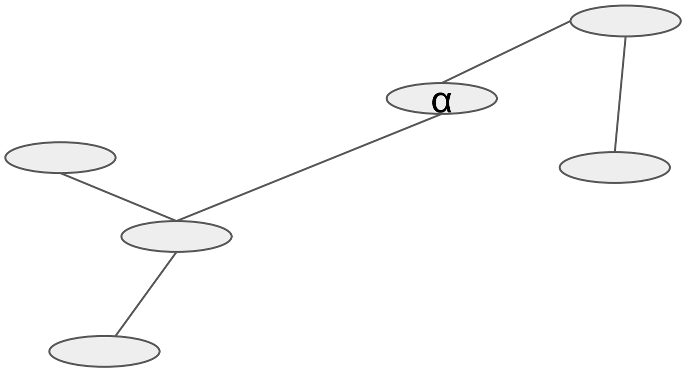
Structuring the unstructured

D() denotes disagreements in OPT
$D(OPT') \leq D(OPT) + D(\alpha)\ (n-1)$
α minimizes disagreements

1) **Find a root.**
2) Find depths of leaves.

$$D(OPT) = \frac{1}{2} \sum_u D(u)$$

OPT



Structuring the unstructured

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1) **Find a root.**
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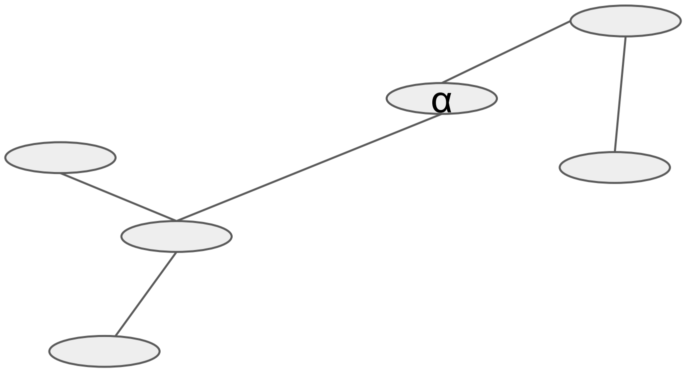
$$D(OPT) = \frac{1}{2} \sum_u D(u)$$
$$\geq \frac{1}{2} \cdot n \cdot D(\alpha)$$

OPT



Structuring the unstructured

D() denotes disagreements in OPT
D(OPT') ≤ D(OPT) + D(α) (n-1)
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1) **Find a root.**
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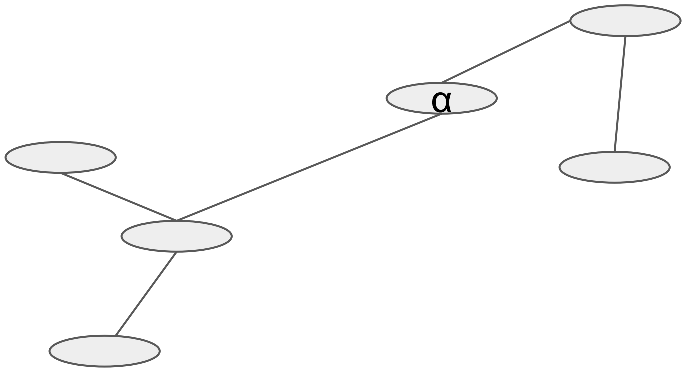
$$D(OPT) = \frac{1}{2}\sum_u D(u)$$
$$\geq \frac{1}{2} \cdot n \cdot D(\alpha)$$

$$D(\alpha) \leq 2D(OPT)/n$$

OPT



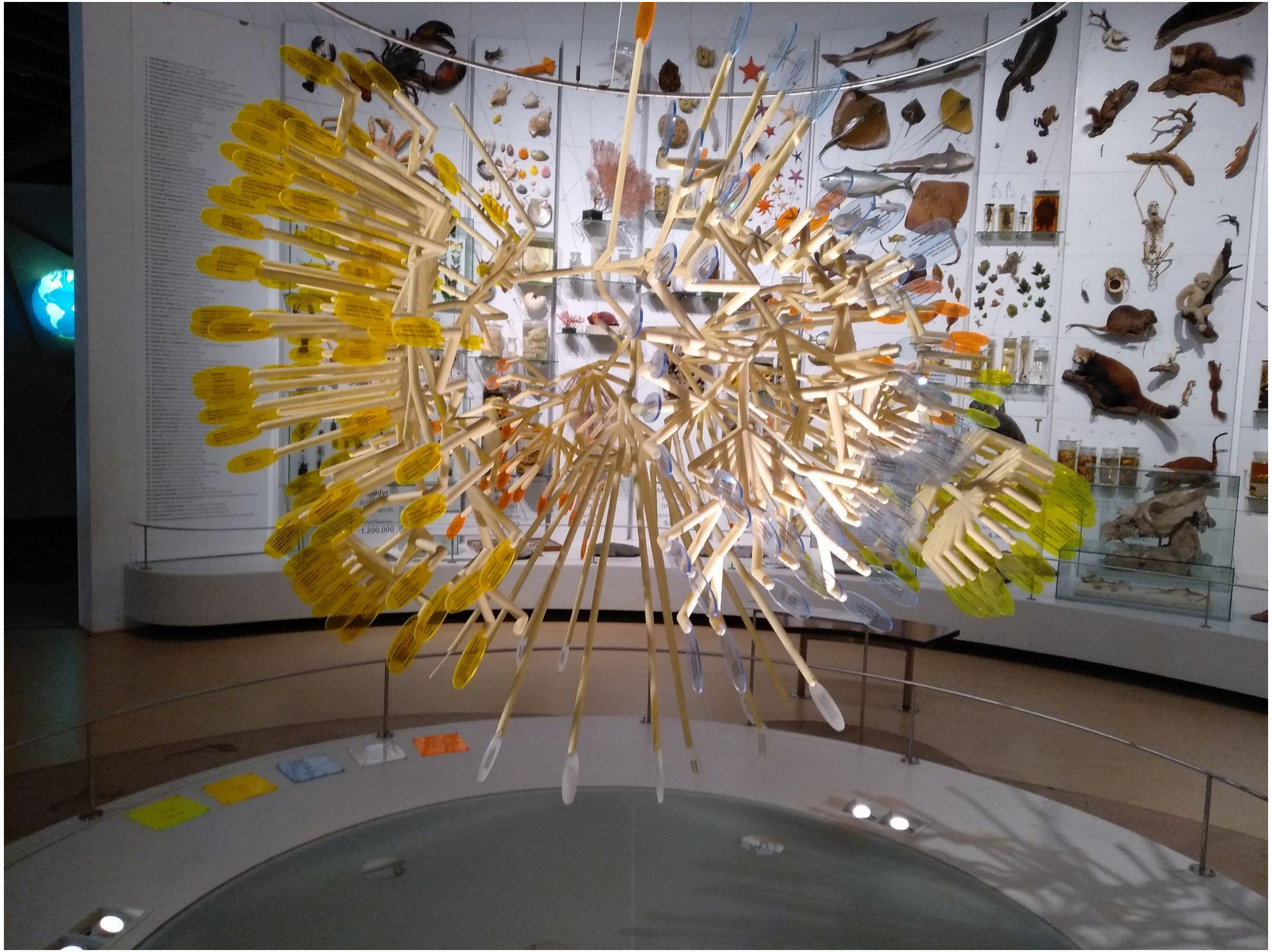
Structuring the unstructured

D(OPT') ≤ D(OPT) + D(α) (n-1)

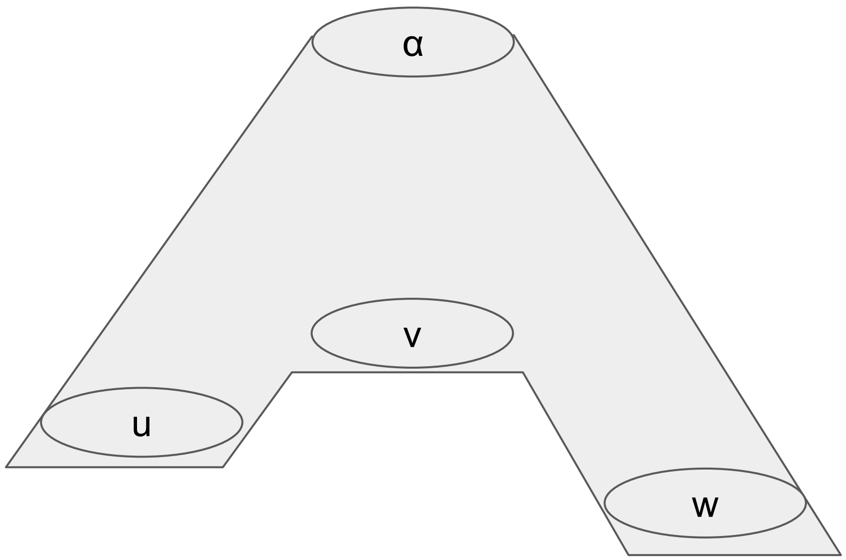
1) **Find a root.**
2) Find depths of leaves.

D(OPT') ≤ 3D(OPT)

$$D(\alpha) \leq 2D(OPT)/n$$



Where do we stand?



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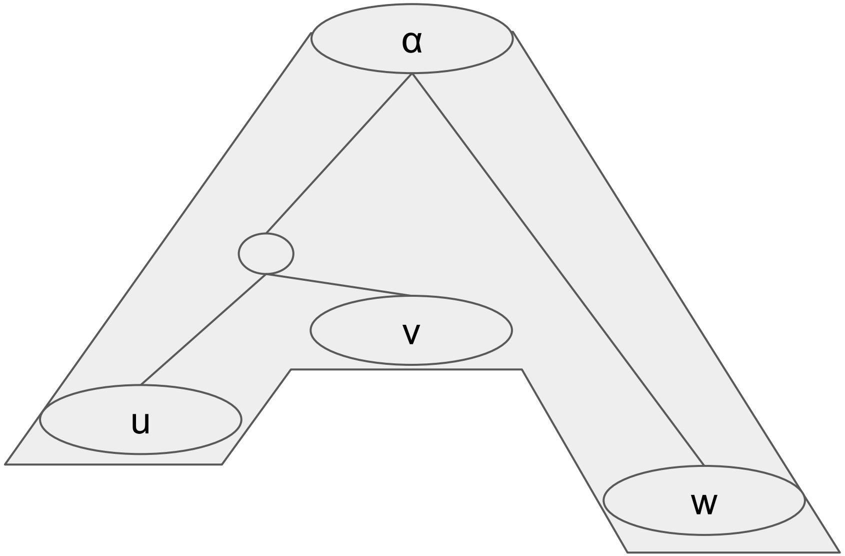

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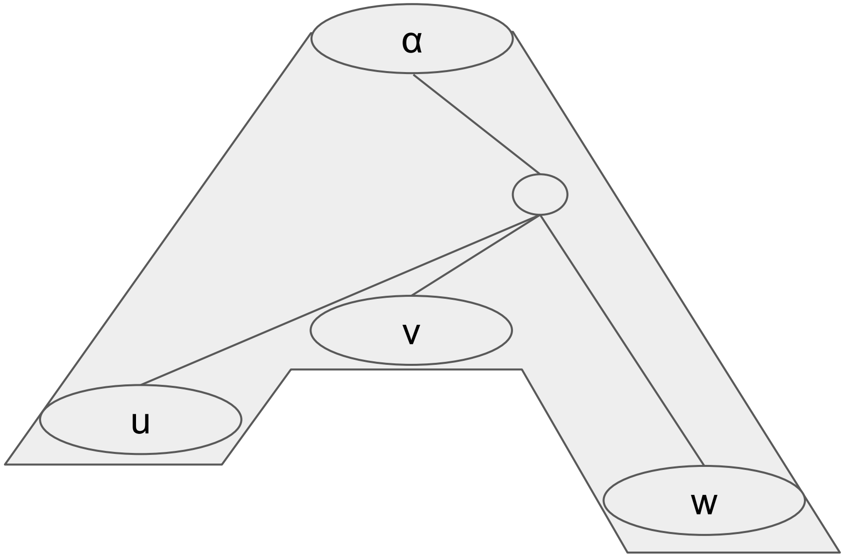

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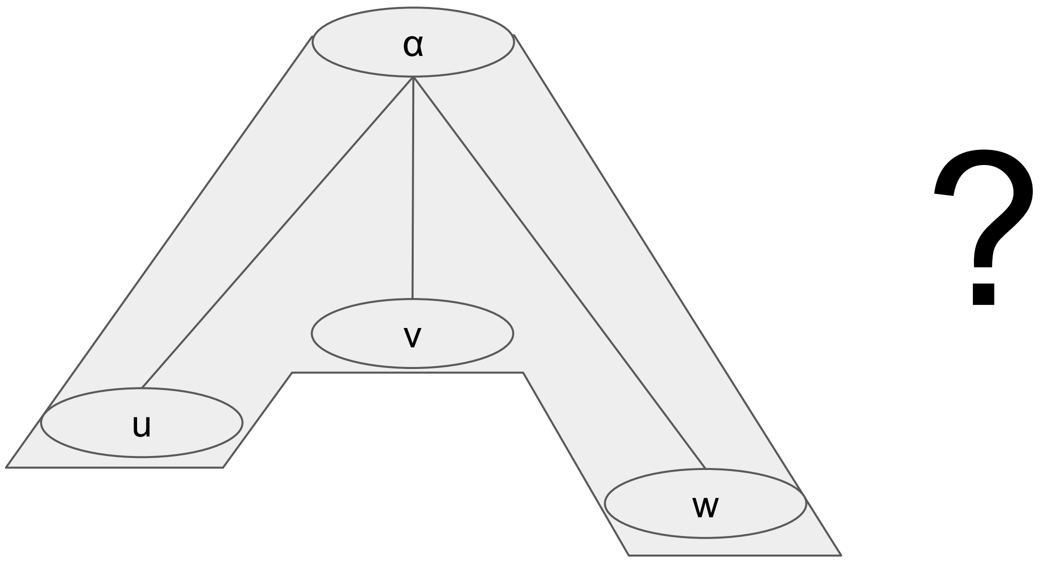
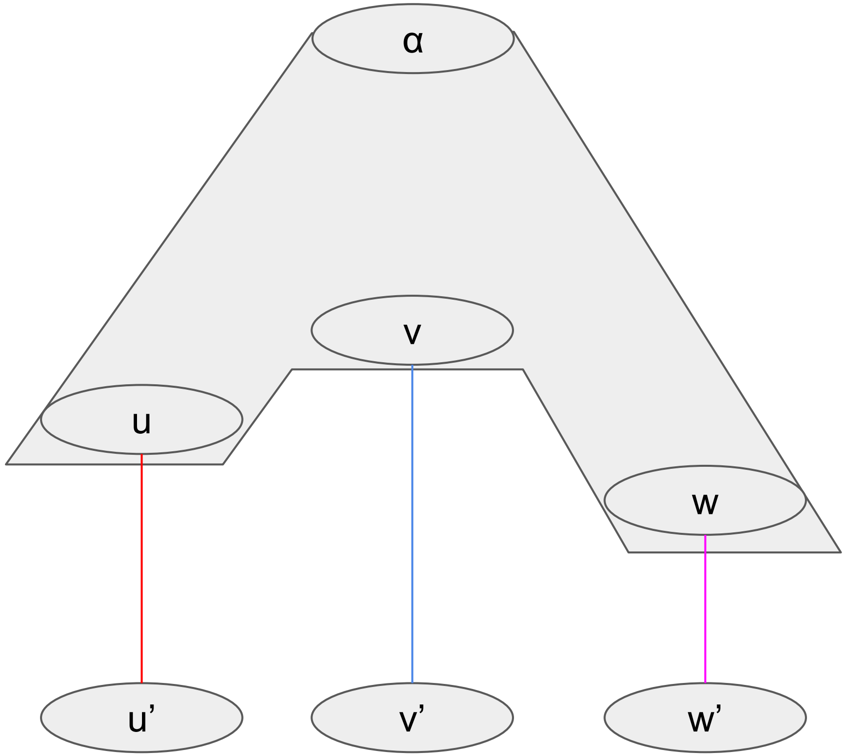


Reduce to ultrametric (all leaves same depth)



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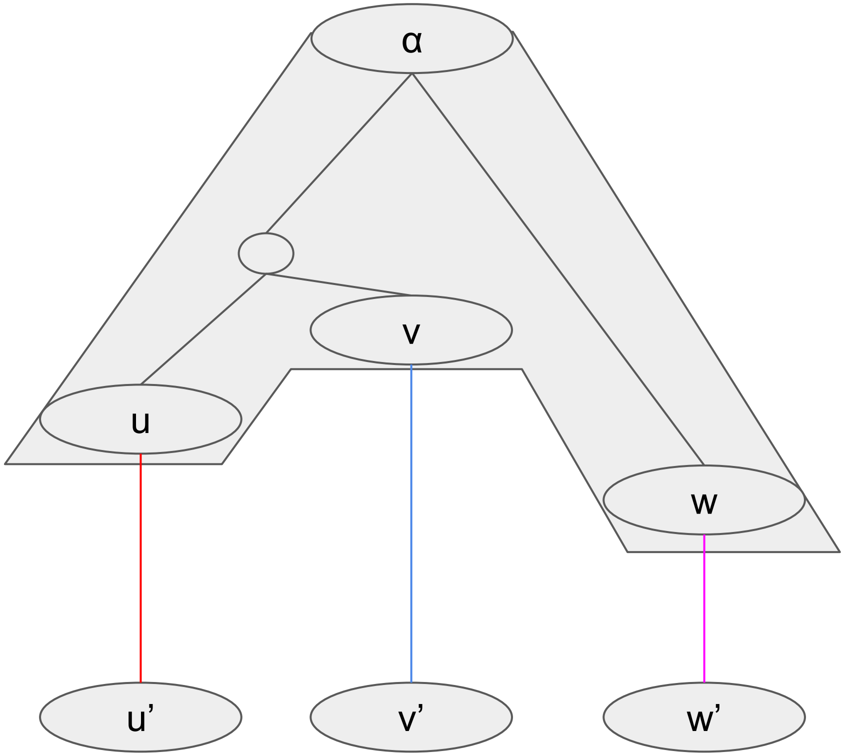

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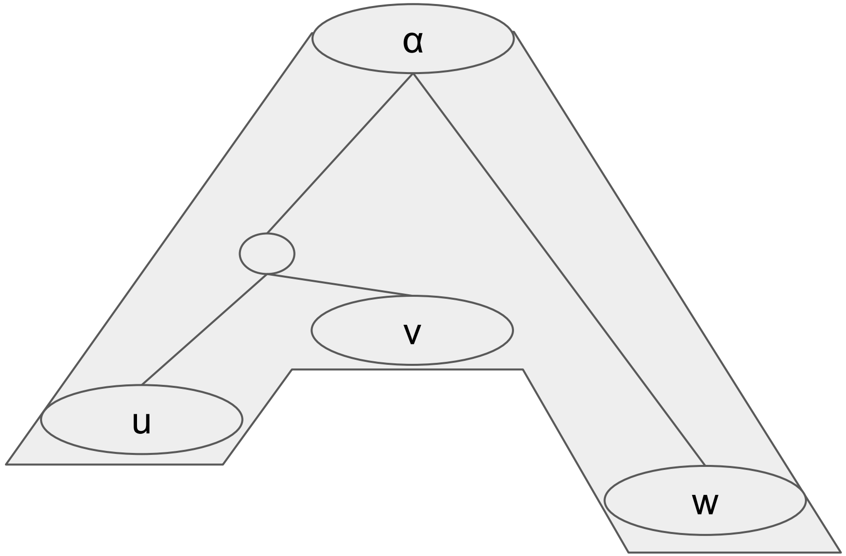

So… who is the “Oldest Sister”?

Was it the sponge or the comb jelly that diverged first?

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