Cost-sharing methods.

Lecture 1. Exercises

1. Show that Pal Tardos cost-shares are cross-monotonic.

2. Prove that the Mulin mechanism with cross-monotonic cost-shares is group-strategyproof. Hint: Consider case i) of a player that is discarded rather than retained in the solution because he lowers his bid. Consider case ii) of a player that is retained in the solution rather than discarded since he raises his bid.

3. Starting from a primal dual monotonically increasing algorithm for set cover, find a truthful algorithm for set cover. Hint: Write an LP formulation and the corresponding dual. Raise uniformly the dual of elements that are not covered. Pick a set whenever it is fully paid. Share the cost of the set between the elements that are covered. Discard players whenever the dual is above the bid. (Scaling the duals by Hn gives a feasible dual)

4. Extend the truthful mechanism to non-metric facility location. Hint: Consider sets for each facility and subset of demands.

5. Show an example where the total dual constructed from KLS is ½ the optimum. Hint: Same example works for undirected cut relaxation.

6. Give a simple proof of 2-budget balance for KLS on Steiner tree. Hint: show that no edge is constructed whenever the optimal tree is connected in one single component.

7. Show that cost-monotonic cost shares provide an allocation in the core.

8. Modify the lower bound for Steiner tree to obtain a lower bound of 3 for Facility location. Hint: place facilities of cost 3 at vertices of f_B, remove connections to the root.