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Cost Sharing and Approximation Algorithms

Exercise 1

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Problem 1. *Consider the cost sharing variant of the* minimum makespan scheduling problem: *We are given a set of n jobs U and a set of m identical machines M. Each job $j \in U$ has a* processing time $p_j > 0$. *A feasible* schedule $\sigma : U \to M$ *assigns each job $j \in U$ to a machine $\sigma(j) \in M$. Let $L_i(\sigma)$ denote the total* load *assigned to machine $i \in M$, i.e.,*

$$L_i(\sigma) = \sum_{j \in U : \sigma(j) = i} p_j.$$

The makespan $C_{\max}(\sigma)$ *of a feasible schedule σ is defined as the maximum load over all machines: $C_{\max}(\sigma) = \max_{i \in M} L_i(\sigma)$. The goal is to compute a feasible schedule σ that minimizes $C_{\max}(\sigma)$.*

Suppose each job corresponds to a player. The cost *for player set $S \subseteq U$ is defined as $C(S) = C_{\max}(\sigma_S^*)$, where σ_S^* is an optimal schedule for the jobs in S.*

(a) Develop a group-strategyproof cost sharing mechanism that is $(2 - \frac{1}{m})$-budget balanced.

(b) Show that there is no Moulin mechanism that is β-budget balanced for $\beta < 2 - \frac{2}{m+1}$.

(c) Adapt the mechanism from (a) such that it is $(H_n + 1)$-approximate.

(d) Show that there is no Moulin mechanism that is α-approximate for $\alpha < H_n$.