ADFOCS 2013

Algebraic Approaches to Exact Algorithms

Exercises, Thursday, 8th August 2013

1. Describe a $O(2^{\omega n/3})$-time algorithm for the Max Cut problem.

   > Max Cut
   > **Input:** Graph $G = (V, E)$, a number $k \in \mathbb{N}$
   > **Question:** Is there a subset $S \subseteq V$ such that there are at least k edges between S and $V \setminus S$?

2. In the *weighted* k-path problem, we are given a directed graph $G = (V, E)$ with a weight function $w : E \to \{0, \ldots, W\}$ and the goal is to find a k-path P of smallest weight (i.e. $\sum_{uv \in E(P)} w(u, v)$). Describe a $O(2^k \cdot W \cdot p(k, n))$-time algorithm for this problem, for some polynomial p.

   Hint: Introduce a new variable.

3. Describe a $O^*(2^{3k})$-time algorithm for the Triangle Packing problem

   > Triangle Packing
   > **Input:** Graph $G = (V, E)$, a number $k \in \mathbb{N}$
   > **Question:** Does G contain k disjoint triangles?

4. In the lecture we have seen a $O^*(2^{3/4k}) = O^*(1.682^k)$-time algorithm for undirected k-path. This can be tuned to $O^*(1.66^k)$. Can you find a possible way of doing it? (Warning: altough the idea is simple, calculating the 1.66 constant can be hard, especially witout the aid of a computer.)

   Hint: Reduce the number of labels, but not for free.

5. Consider the following problem:

   > Colorful Graph Motif
   > **Input:** Graph $G = (V, E)$, a coloring $c : V \to C$, a set of colors M
   > **Question:** Is there a subset $S \subseteq V$ such that $G[S]$ is connected, and $c(S) = M$?

   Denote $k = |M|$. Describe a $O^*(c^k)$-time algorithm for a constant c, ideally a $O^*(2^k)$-time algorithm.

   You can also consider a (slightly different, perhaps slightly simpler) problem, where the vertices in the subset S must form a path.