Fine-grained Complexity — Exercises

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Solve the following problems.

Problem 1: Subquadratic algorithms for 3-sum

Let a_i, b_i and c_i for $1 \leq i \leq n$ be three lists of integers in the range $[0, d]$. Find an algorithm that runs in time $O((n + d) \log(nd))$ to determine whether there exist i, j and k such that $a_i + b_j = c_k$. You can assume that the basic arithmetic operations on the integers in the range $[0, d]$ can be performed in unit time.
Hint: Use Fast Fourier Transform.

Problem 2: Variations of the 3-sum problem

Consider the following variation of the standard 3-SUM problem.
Tripartite 3-sum: Let a_i, b_i and c_i for $1 \leq i \leq n$ be three lists of integers in the range $[0, d]$. Determine whether there exist i, j and k such that $a_i + b_j = c_k$.

Prove that (3-SUM, n^2) and (TRIPARTITE 3-SUM, n^2) are fine-grained reducible to each other.

Standard 3-SUM is problem is defined below.
3-sum: Given a sequence of integers $x_1, x_2, \ldots, x_n$ where $x_i \in [0, 1, \ldots, d]$, do there exist i, j and k such that $x_i + x_j = x_k$?

Problem 3: Collinear points

Show that 3-SUM can be reduced to the following problem so that a polynomial improvement in solving the problem would result in an algorithm for 3-sum with polynomial improvement over n^2.

Given a set of points in the plane with integer coordinates, is there a line that contains at least 3 points?

Problem 4: Orthogonal Vectors problem in low dimension

Show that the ORTHOGONAL VECTORS problem for n vectors of dimension $c \log n$ can be solved in time n^{c+1}.

Problem 5: Minimum Hamming Distance problem

Assuming **SETH**, show that for any $\varepsilon > 0$ there is a c such that solving the minimum distance problem on $d = c \log n$ dimensions requires time $\Omega(n^{2-\varepsilon})$.

Minimum Hamming Distance problem: For any d and l, decide if two input sets $U, V \subseteq \{0,1\}^d$ with $|U| = |V| = n$ have a pair $u \in U$ and $v \in V$ such that $\|u - v\|_2^2 < l$.