Problem sets ADFOCS 2024

Topics in mechanism design

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Set 1

1. Consider the problem of related machines scheduling in which machines are agents with private values for their speeds $s_1, \ldots, s_n$, and we want to minimize the makespan. Consider the mechanism that selects an arbitrary optimal solution. Is this mechanism truthful for 2 machines? For 3 machines?

2. Give a polynomial-time truthful deterministic mechanism with an approximation ratio $3/2$ for the related scheduling problem with 2 machines. Recall that in the related scheduling problem there are two machines with speeds (s_1, s_2), the input is a set of jobs $\{w_1, \ldots, w_n\}$ and we want a monotone mechanism to minimize the makespan.

3. Let's consider a two-sided auction. There are two agents: a seller and a buyer with valuations (v_1, v_2), respectively, for a single item. Assume that agent 1 owns the item.

   We want a mechanism that trades the item, i.e., it decides whether to leave the item to agent 1 or move it to agent 2.

   Describe the VCG mechanism with Clarke pivot rule for this setting. What are the payments? Does the mechanism have the "budget-balance" property?

   Analyze the situation when there is another buyer with value v_3.

4. Consider the case of a simple item to be allocated to one of n agents with nonnegative values $v_1, \ldots, v_n$. A mechanism is truthful for an agent i if and only if their payment p_i depends only on the values v_{-i} of the other agents.

   (a) Suppose that $p_i(v_{-i})$ are arbitrary functions. Explain why this mechanism may not be valid.

   (b) Suppose that there are only two agents and that the mechanism is defined by a function $p_1(v_2)$ as follows: if $v_1 \geq p_1(v_2)$, the item is allocated to agent 1, otherwise it is allocated to agent 2. What property must $p_1(v_2)$ satisfy for the mechanism to be truthful?

   (c) Let's generalize the mechanism to multiple agents: there are payment functions $p_1(v_{-1}), \ldots$, $p_{n-1}(v_{-(n-1)})$, $p_n(v_{-n}) = 0$ and the mechanism allocates the item to the first agent i with $v_i \geq p_i(v_{-i})$. For which functions $p_i(v_{-i})$ is the mechanism truthful?

5. Consider the following mechanism for selling $k > 1$ identical items to unit-demand bidders. The bidders with the k highest bids get an item and pay as follows: The highest bidder pays the second highest bid, the second highest bidder pays the third highest bid, etc. Is this mechanism truthful for all bidders?

Set 2

1. Consider the scheduling problem with two unrelated machines and two tasks. VCG for this setting tries to minimize the social cost, i.e., it computes

$$\arg\min(t_{1,1} + t_{1,2},\, t_{1,1} + t_{2,2},\, t_{2,1} + t_{1,2},\, t_{2,1} + t_{2,2}),$$

   and gives the two tasks accordingly (for example, if the minimum comes from the third value $t_{2,1} + t_{1,2}$, machine 1 gets the second task and machine 2 gets the first task). Now consider changing the first expression from $t_{1,1} + t_{1,2}$ to $t_{1,1} + t_{1,2} - 1$. Argue that this mechanism is an affine minimizer. Show the partition of the space into allocations of the first machine, when the second machine has values $t_2 = (3, 2)$. Show that this mechanism has unbounded approximation ratio, when the objective is the makespan.

2. Consider the scheduling problem with two unrelated machines and two tasks. Consider an affine minimizer with $\lambda_i = 1$ for $i = 1, 2$. What are the conditions on payments (or equivalently on γ's in the definition of affine minimizers) so that the mechanism is quasi-bundling? quasi-flipping? task-independent?

3. Analyze the approximation ratio of the Hybrid mechanism for a star of 2 leaves.

   Can you suggest another truthful mechanism with better approximation ratio?

4. Recall the SQUARE mechanism, which is a fractional task-independent mechanism that allocates fractions inversely proportional to the square of the values. Prove that it has approximation ratio $(n + 1)/2$.