

Fat Curves – A title which is much too long to serve as running head

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Abstract

Fat curves in two-dimensional Euclidean space are discussed. Previous work on fat curves is reviewed and a new definition is given for a fat curve having a smooth axis. The joining of two fat curves is discussed and a technique for scan-converting fat curves is presented.

Note: This article has been modified to demonstrate the use of \LaTeX and the EG Publication Style file.

Categories and Subject Descriptors (according to ACM CCS): I.3.3 [Computer Graphics]: Line and Curve Generation

1. Introduction

In computer graphics it is frequently desirable to represent a curve or the outline of an object as “thick” or a “fat” curve. This is particularly true for high-resolution graphics devices where curves represented by a chain of single pixels are too faint or where for aesthetic reasons the curve should have a fixed non-zero width. The definition of such objects pose particular geometric problems. Implementing the objects in terms of raster graphics similarly introduces further problems in the scan-conversion process.

If we look to standard references on algorithms for computer graphics such as Pavlides¹ then we do not find the fat line or curve concept. In fact there are only two fundamental concepts considered by Pavlides¹: a thin curve with an orientation-dependent average width ranging from $1/\sqrt{2}$ pixels to 1 pixel and a full region. Fat curves are subsumed under the full region concept and no consideration is given to the particular problems encountered in dealing with them.

We want here only show a possible citation, such as the typical citation of the Foley et al. book² or a well known paper on ray tracing of volumetric dataset⁵. Fat lines are discussed in Bresenham under the concept of *Widelines* and he poses a number of questions relating to this concept. A fundamental question is how the fat lines are terminated and what the assumptions are when such lines are joined at decreasing angles. To quote Bresenham:

Is *Wideline* a consistent concept, or is it a poorly specified and incompletely defined attempt to set up an implicit but fuzzily understood reference

model of areas in contrast to lines? What is the shape of wideline ends? Is line width a geometric property in our original modeling coordinate space, or is such thickness only a picture-rendering cosmetic attribute akin to pseudo-pen size in final raster space? How should projective transformations affect *Widelines*? If width is a geometric attribute, what is the implied boundary definition?

In this paper we discuss some of the problems posed by Bresenham and we suggest solutions both in the underlying geometric setting and in the raster plane. We first give a precise definition of a fat line or curve as a continuous geometric object. Then, using this definition, we develop new algorithms implementing scan-conversion for such curves.

In the next section we survey previous definitions of fat lines concluding with the specific problems that we attempt to solve in this paper. In Section 4 we consider the analytic definition of smooth fat curves and we verify some simple properties. The problem of joining fat curves is then dealt with and we introduce the concept of a piece-wise smooth fat curve. Finally we give a method for the scan-conversion of the fat curves we defined in the previous sections.

2. Previous Work

Fat lines are discussed in several recent papers, but here we cite papers that have nothing in common with the *fat line* topic^{5,4,6,7}. The concept is also used by several advanced

The DoE/SDD boundary data contain some gross digitising errors. For example, inlet X in Figure 2c does not feature on conventional Ordnance Survey 1: 50 000 maps of the area. The data are also not very accurate where coastlines are convoluted. Even if we ignore these and other gross errors, such as spikes, there will always be an element of random error in digitised data. It is reasonable to assume that points digitised from 1: 50 000 source material may only be accurate to within ± 5 metres. This algorithm does not lead to a substantial accumulation of rounding errors, hence the numerical errors discussed earlier tend to be very small compared with digitising errors.

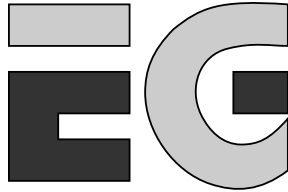


Figure 2: Another way of embedding a PS file by explicitly specifying the bounding box.

For the purposes of our argument, it is unnecessary to undertake an exhaustive evaluation of the consequences Douglas and Peucker have treated overhangs and closed loops as different problems, and have used different methods to cope with each case.

4.1. Numerical Problems

The FORTRAN programs by Douglas, White, and Wade use single precision REALS when computing offsets (see results in Table 1). Whilst double precision accuracy may be attained through the use of compiler options, we are unsure whether previous research has been based on programs compiled in this manner. Wade's program was so compiled for use in our previous evaluations. Forrest stated that Ramshaw (1982) had to adopt carefully tuned double and single precision floating point arithmetic to compute the intersection of line segments whose end points were defined as integers. Forrest exclaimed "This is an object lesson to us all: constructing geometric objects defined on a grid of points, requiring ten bits for representation can lead to double precision floating point arithmetic!"

Most evaluative studies do not cite the co-ordinates in use. We do not know whether the published test lines were in original digitiser co-ordinates or whether they had been converted to geographic references. British National Grid co-ordinates for the administrative boundaries of England, Scotland and Wales (digitised by the Department of Environment (DoE) and Scottish Development Department (SDD)) are input to one metre accuracy and require seven decimal digits for representation if we include the northern islands

of Scotland. At the South West Universities Regional Computer Centre these co-ordinates have been rounded to 10 metre resolution; even this requires six decimal digits. Seamless cartographic files at continental and global scales use much larger ranges of geographic co-ordinates.

Machine	Points	Calculated squares of offset values	
		Single Precision	Double Precision
ICL 3980			
	(C)	28199.351562500	28143.490838958
	(D)	28171.789062500	28143.490838961
VAX 8200			
	(C)	28253.095703125	28143.490838958
	(D)	28165.806640625	28143.490838958
SEQUENT SYMMETRY			
	(C)	28145.100000000	28143.490838961
	(D)	28145.100000000	28143.490838961
SUN 3/60			
	(C)	28253.095703125	28143.490838961
	(D)	28165.806640625	28143.490838961

NOTES

Offsets of points C and D from the anchor-floor line A–B as calculated using Wade's program. Points A, B, C and D are shown in Figure 5. The British National Grid coordinates (in metres) of the points are as follows:

Point A	238040 (x1)	205470 (y1)	ANCHOR
Point B	237890 (x2)	205040 (y2)	FLOATER
Point C	237810 (x3)	205320 (y3)	
Point d	238120 (x3)	205190 (y3)	

Note that the above co-ordinates may be used in conjunction with the expression presented in section 3.2.2a to check the tabulated results.

Table 1: The Precision of Calculations

A limited number of papers actually described improved for new algorithms or methods for visualization^{5, 6, 7}. This may be caused by the complexity of the environment in which a method is used; issues of system architecture, user interface, data handling, etc. must be dealt with before a new presentation technique can show its full advantage. But even so, we think the field can use more contributions of this type.

There was also a discussion session on the merits of animation and special effects (such as sound) to support visualization. For example, in the area of flow visualization, it is quite common to use animation, and techniques for video registration have been developed.

5. Issues in Visualization

Scientific visualization is an interdisciplinary field, which can only flourish when computer graphics experts cooperate with specialists from application areas, and providers of computing, visualization, and data management facilities. Therefore, it is essential that all of these viewpoints are represented in research projects and also in meetings such as

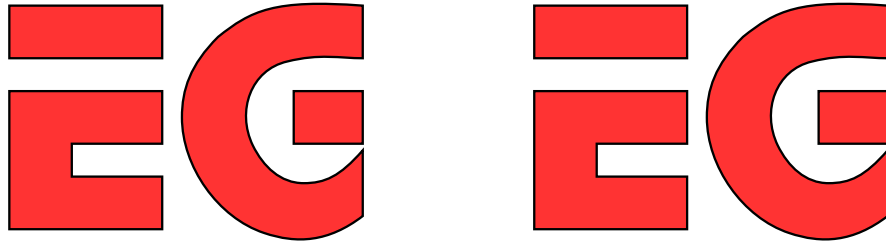


Figure 3: Here are two sample color figures.

MIND: for the pinte~~d~~ version – and **ONLY** for the printed version – color figures have to be placed in the last page. For the electronic version, which will be converted to PDF before making it available electronically, the color images should be embedded within the document. Optionally, other multimedia material may be attached to the electronic version.