BDI
A New Decidable First-Order Clause Class

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Motivation

Since the beginning of the 20th century there is an interest in decidable first-order classes:

- Bernays-Schönfinkel: $\exists^*\forall^*\phi$
- Ramsey: $\exists^*\forall^*\phi$ + Equality
- Ackermann: $\exists^*\forall\exists^*\phi$
- Gödel, Kalmar, Schutte: $\exists^*\forall^2\exists^*\phi$

Decidable Clause Classes:

- $\mathcal{PVD}$: $P(f(x,y)), Q(y) \rightarrow R(x,y)$
- Guarded Fragment: $Q(y), R(x,y) \rightarrow P(g(x))$

Motivation for BDI (Bounded Depth Increase)

Modelling Business Processes:

$$\mathrm{User}(y), \boxed{\mathrm{BusReq}(x)}, \mathrm{Auth}(y, \mathrm{PurchReq}) \rightarrow \boxed{\mathrm{ToBeReleased}(\mathrm{preq}(y, x))}$$

$$\mathrm{User}(y), \boxed{\mathrm{ToBeReleased}(\mathrm{preq}(y, x))}, \mathrm{Auth}(y, \mathrm{PurchRel}) \rightarrow \boxed{\mathrm{Released}(\mathrm{preq}(y, x))}$$

$$\vdots$$

Modelling Business Process Properties:

$$\boxed{\mathrm{Released}(\mathrm{preq}(y, x))} \rightarrow \boxed{\mathrm{BusReq}(x)}$$

The Clauses do not Belong to any Known Decidable Clause Class.

Undecidability is Close

Post Correspondence Problem: $(aa, ba, bba) \quad (a, aba, abb)$

Solution: 1,3,2 $aabbaba$

Straight Forward PCP Encoding: $(u_1, u_2, \ldots, u_k) \quad (v_1, v_2, \ldots, v_k)$

code words by monadic functions: aba coded by $f_a(f_b(f_a(x)))$

abbreviate $f_a(f_b(f_a(x)))$ by $f_{aba}(x)$

then the overall problem is coded by

$$
\begin{aligned}
&\rightarrow P(f_{u_i}(a), f_{v_i}(a)) \\
P(x, y) &\rightarrow P(f_{u_i}(x), f_{v_i}(y)) \\
P(x, x) &\rightarrow
\end{aligned}
$$

Undecidability is Close

Straight Forward PCP Encoding: $(u_1, u_2, \ldots, u_k) \quad (v_1, v_2, \ldots, v_k)$

$$
\begin{aligned}
&\rightarrow P(f_{u_i}(a), f_{v_i}(a)) \\
P(x,y) &\rightarrow P(f_{u_i}(x), f_{v_i}(y)) \\
P(x,x) &\rightarrow
\end{aligned}
$$

Contained in BDI:

- recursive definition of P
- depth increasing clauses

Forbidden by BDI:

- more than one depth increasing position
- unlimited depth increasing derivations

A Sketch of BDI

$P(x, g(y)), Q(z) \to R(x, x), Q(y)$ $\qquad$ $P(x, y), Q(z) \not\to R(x, g(x)), Q(v)$

variables right must occur left, no increase ($\mathcal{PVD}$)

$P(x, y), Q(z) \to R(x, f(y, g(x))), Q(y)$ $\qquad$ $R(x, y) \not\to R(x, g(x)), Q(y)$

all R atoms are of shape $R(*, f(*, g(*)))$

$P(x, y), Q(z) \to R(x, g(x)), Q(y)$ $\qquad$ $R(x, y) \not\to R(x, g(y)), Q(y)$

no depth increase in P atoms and R first argument

Hyper-Resolution on BDI

$$\begin{aligned} &\rightarrow P(g(a), a) \\ &\rightarrow Q(a) \\ P(x, y), Q(z) &\rightarrow R(x, f(x, z)), Q(y) \end{aligned}$$

yields $\rightarrow R(g(a), f(g(a), a)), Q(a)$

in general

- hyper resolvents are ground
- depth bounded by $2 *$ max clause depth

but prolific

$\rightarrow P(t_1, s_1) \quad \ldots \quad \rightarrow P(t_n, s_n)$

$\rightarrow Q(r_1) \quad \ldots \quad \rightarrow Q(r_m)$

yield already $n * m$ hyper-resolvents in one step

Ordered Resolution on BDI

restrict resolution to maximal literals and selection

$$\begin{aligned} &\rightarrow P(g(a), a) \\ &\rightarrow Q(a) \\ P(x, y), Q(z) &\rightarrow \boxed{R(x, f(x, z))}, Q(y) \end{aligned}$$

no inference needed at all

in general termination of ordered resolution through

- cutting off clauses that violate depth bound
- condensing clauses that contain too many variables

Summary

- new decidable clause class BDI
  - BDI enables recursive definitions
  - BDI enables depth increase
  - BDI enables non-linear atoms and terms
- decidable by hyper-resolution, but prolific
- decidable by ordered resolution, works well in practice
- successfully tested on SAP authorization problems
  - thousands of users
  - thousands of authorizations

Thanks for your attention!